# Inverse Function Diagnostic Quiz

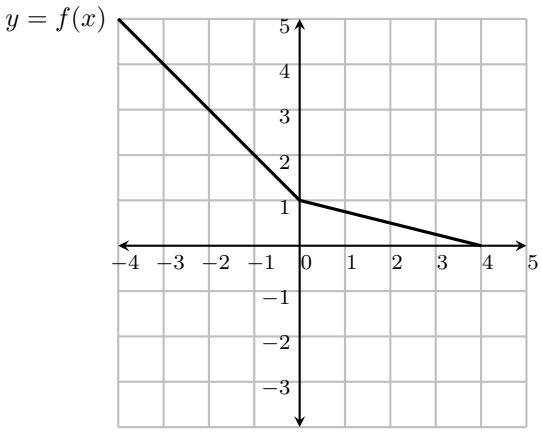
Take this quiz to see if you need Lecture 4 (Inverse Functions). Answers are on page 2.

**Important:** Pencil or pen only. **No calculators**.

1. Does the function $f(x) = x^2 + 2x + 1$ have an inverse? If so, find its inverse.

2. Does the function $f(x) = \sqrt[3]{x-1}$ have an inverse? If so, find its inverse.

3. Does the function graphed below have an inverse? If so sketch the graph of $y = f^{-1}(x)$.



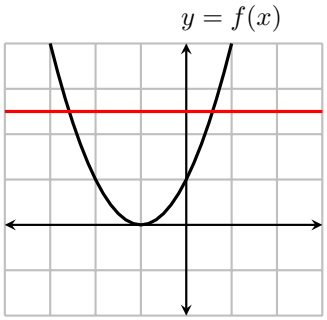

4. Suppose that $f$ is an invertible function. Simplify $f^{-1}\left(f(x^3 + 5x)\right)$.

Here are the solutions. If your answers are not all correct, then you probably need Lecture 4.

1. Does the function $f(x) = x^2 + 2x + 1$ have an inverse? If so, find its inverse.

   **No.** The function $f(x) = x^2 + 2x + 1 = (x+1)^2$ is a parabola, as shown below.

   A horizontal lines crosses the graph more than once, so $\boxed{f(x) \text{ has no inverse}}$.




2. Does the function $f(x) = \sqrt[3]{x-1}$ have an inverse? If so, find its inverse.

   The graph of this function is the graph of $y = \sqrt[3]{x}$ moved one unit to the right.

   No horizontal line crosses the graph more than once, so the inverse exists.

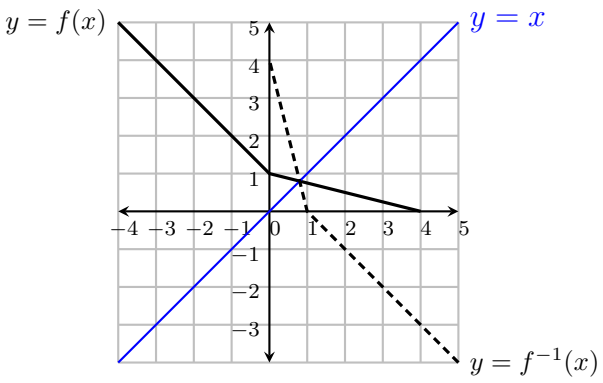
   Let's now compute the inverse.

$$\begin{aligned} y &= \sqrt[3]{x-1} \\ x &= \sqrt[3]{y-1} && \text{(interchange variables)} \\ x^3 &= \sqrt[3]{y-1}^3 && \text{(solve for } y\text{)} \\ x^3 &= y - 1 \\ y &= x^3 + 1 \end{aligned}$$

   Therefore $\boxed{f^{-1}(x) = x^3 + 1}$.

3. Does the function graphed below have an inverse? If so sketch the graph of $y = f^{-1}(x)$.

   **Yes.** No horizontal line crosses the graph of $y = f(x)$ more than once, so this function is one-to-one and therefore has an inverse. The graph $y = f^{-1}(x)$ of the inverse (shown dashed below) is the graph of $y = f(x)$ reflected across the line $y = x$.




4. Suppose that $f$ is an invertible function. Simplify $f^{-1}(f(x^3 + 5x))$.

   $f^{-1}(f(x^3 + 5x)) = \boxed{x^3 + 5x}$.