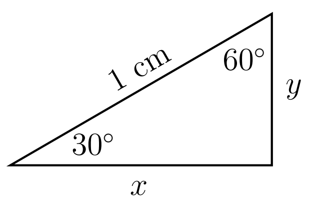
Trigonometry Diagnostic Quiz

Take this quiz to see if you need Lectures 3A and 3B on trigonometry. Answers are on page 2.

Important: Pencil or pen only. **No calculators**.

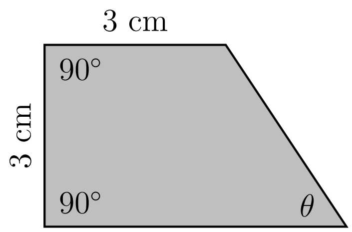
1. Find $\sin(7\pi/4)$

2. Find the missing side lengths x and y of this right triangle.
   (You may be able to do this without using trigonometry.)



3. Convert 120° to radians.

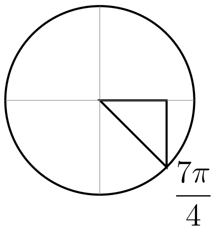
4. Find the area of this shape. (Your answer will depend on the angle θ and involve a trig function.)



5. Find **all** solutions to the equation $x\cos(x) = -x$.

Here are the solutions. If your answers are not all correct, then you probably need Lectures 3A and 3B.

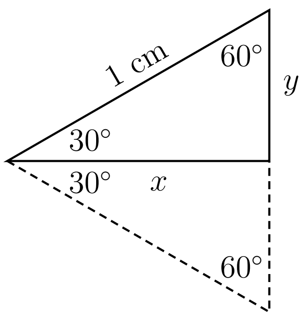
1. $\sin(7\pi/4) = -\dfrac{\sqrt{2}}{2}$. This answer should be immediate. If it is not, you can work it this way:



Solution: Locate $7\pi/4$ on the unit circle.
It forms a familiar 45-45-90 triangle (with hypotenuse 1).
From your knowledge of this triangle, $\sin(\frac{7\pi}{4}) = -\frac{\sqrt{2}}{2}$.
(Answer is negative because triangle is below the x-axis.)

2. Find the missing side lengths x and y of this right triangle.

Note: Ideally this triangle is familiar, and the answers are obvious. If not, here is a solution.

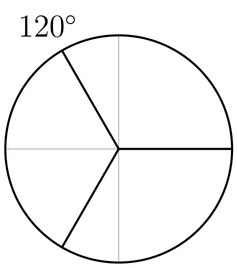


The triangle is the upper half of an equilateral triangle with side length 1, as shown. So y is half of 1, that is, $\boxed{y = 1/2}$.

From this, we can find x with the Pythagorean theorem:

$$\begin{aligned} x^2 + (1/2)^2 &= 1^2 \\ x^2 + 1/4 &= 1 \\ x^2 &= 3/4 \end{aligned} \qquad \text{So } \boxed{x = \sqrt{3}/2}.$$

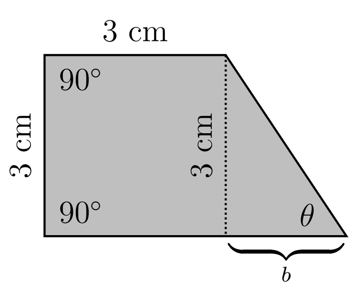
3. Convert 120° to radians.



Solution: 120° is one third of the circumference of the unit circle. Because the circumference of the unit circle is 2π, we know that 120° is $\frac{1}{3} \cdot 2\pi = \boxed{\frac{2\pi}{3} \text{ radians}}$.

(But if your trigonometry skills are good, you will see this immediately, without having to work it out.)

4. Find the area of this shape. (Your answer will depend on the angle θ and involve a trig function.)



The region is made up of a 3×3 square, and a triangle.
The height of the triangle is 3. Denote its base as b.
To find b, notice $\dfrac{b}{3} = \dfrac{\text{ADJ}}{\text{OPP}} = \cot(\theta)$, so the base is $b = 3\cot(\theta)$.
Thus $\text{AREA} = 3 \cdot 3 + \frac{1}{2} \cdot 3\cot(\theta) \cdot 3 = \boxed{9 + \frac{9}{2}\cot(\theta) \text{ square cm.}}$

5. Find **all** solutions to the equation $x\cos(x) = -x$.

Write this as $x\cos(x) + x = 0$, which factors as $x(\cos(x) + 1) = 0$. Thus either $x = 0$ or $\cos(x) = -1$. From the unit circle we know that the values of x for which $\cos(x) = -1$ are the odd multiples of π.

Therefore the solutions of this equation are $\boxed{x = 0 \text{ and } x = \pm\pi, \pm 3\pi, \pm 5\pi, \pm 7\pi, \dots}$.