Name: Richard

QUIZ 11 ♡

MATH 200

October 6, 2021

1. Find the derivative: $y = \dfrac{1}{x^2+\ln(x)}$ $= (x^2+\ln(x))^{-1}$

$$y' = -1\cdot(x^2+\ln(x))^{-1-1} D_x\left[x^2+\ln(x)\right] = -(x^2+\ln(x))^{-2}\left(2x+\tfrac{1}{x}\right)$$

$$= \boxed{-\frac{2x+1/x}{(x^2+\ln(x))^2}}$$

2. Find the derivative: $y = \ln\big(\cos(x)\big)$

$$y' = \frac{1}{\cos(x)} D_x[\cos(x)] = \frac{1}{\cos(x)}(-\sin(x)) = \boxed{-\frac{\sin(x)}{\cos(x)}}$$

$$= \boxed{-\tan(x)}$$

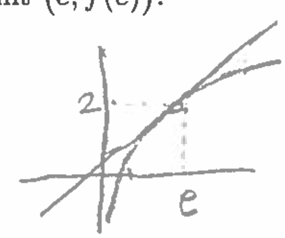
3. Find the derivative: $y = \cos\big(\ln|x|\big)$

$$y' = -\sin(\ln|x|)\, D_x[\ln(x)] = -\sin(\ln|x|)\frac{1}{x}$$

$$= \boxed{-\frac{\sin(\ln|x|)}{x}}$$

4. Find the equation of the tangent line to the graph of $f(x) = 1+\ln(x)$ at the point $\big(e, f(e)\big)$.

Slope at $(x, f(x))$ is $f'(x) = 0 + \frac{1}{x} = \frac{1}{x}$



Slope at $(e, f(e))$ is $f'(e) = \frac{1}{e}$

Point on tangent: $(e, f(e)) = (e, 1+\ln|e|) = (e, 1+1)$
$= (e, 2)$

Point-slope formula:
$y - y_0 = m(x - x_0)$
$y - 2 = \frac{1}{e}(x - e)$ $\longrightarrow$ $y - 2 = \frac{1}{e}x - 1$

$$\boxed{y = \frac{1}{e}x + 1}$$

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QUIZ 11 ♣

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1. Find the derivative: $y = \ln(x^3 + x)$

$$\begin{cases} y = \ln(u) \\ u = x^3 + x \end{cases}$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{u}(3x^2+1) = \frac{1}{x^3+x}(3x^2+1) = \boxed{\frac{3x^2+1}{x^3+x}}$$

2. Find the derivative: $y = \sin\Big(\ln|x|\Big)$

$$y' = \cos(\ln|x|)\,D_x\big[\ln|x|\big] = \cos(\ln|x|)\frac{1}{x}$$

$$= \boxed{\frac{\cos(\ln|x|)}{x}}$$

3. Find the derivative: $y = \dfrac{x \ln|x|}{3x+1}$

$$y' = \frac{D_x[x\ln|x|](3x+1) - x\ln|x|\,D_x[3x+1]}{(3x+1)^2}$$

$$= \frac{\left(1\cdot\ln|x| + x\frac{1}{x}\right)(3x+1) - x\ln|x|\cdot 3}{(3x+1)^2}$$

$$= \boxed{\frac{(\ln|x|+1)(3x+1) - 3x\ln|x|}{(3x+1)^2}}$$

4. Find the equation of the tangent line to the graph of $f(x) = \ln(x)$ at the point $(1/e\, f(1/e))$.

Slope of tangent to $y=f(x)$ at $(x, f(x))$ is $f'(x) = \frac{1}{x}$.

Slope of tangent to $y=f(x)$ at $\left(\frac{1}{e}, f\left(\frac{1}{e}\right)\right)$ is $f'\left(\frac{1}{e}\right) = \frac{1}{1/e} = \boxed{e}$

Point on tangent: $\left(\frac{1}{e}, f\left(\frac{1}{e}\right)\right) = \left(\frac{1}{e}, \ln\left(\frac{1}{e}\right)\right) = \left(\frac{1}{e}, -1\right)$

By point-slope formula the tangent has equation

$$y - y_0 = m(x - x_0)$$

$$y - (-1) = e\left(x - \frac{1}{e}\right)$$

$$y + 1 = ex - 1$$

$$\boxed{y = e\cdot x - 2}$$

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QUIZ 11 ◊

MATH 200

October 6, 2021

1. Find the derivative: $y = \dfrac{e^{-2x}}{x^2+\ln(x)}$

$$y' = \frac{D_x\left[e^{-2x}\right](x^2+\ln(x)) - e^{-2x}\,D_x\left[x^2+\ln(x)\right]}{(x^2+\ln(x))^2}$$

$$= \frac{e^{-2x}(-2)(x^2+\ln(x)) - e^{-2x}\left(2x+\frac{1}{x}\right)}{(x^2+\ln(x))^2} = \boxed{\frac{-e^{-2x}\left(2x^2+2\ln(x)+2x+\frac{1}{x}\right)}{(x^2+\ln(x))^2}}$$

2. Find the derivative: $y = \ln\left(\tan(x)\right)$ $\begin{cases} y = \ln(u) \\ u = \tan(x) \end{cases}$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{u}\sec^2(x) = \frac{1}{\tan(x)}\sec^2(x) = \boxed{\frac{\sec^2(x)}{\tan(x)}}$$

3. Find the derivative: $y = \tan\left(\ln|x|\right)$

$$y' = \sec^2(\ln|x|)\,D_x\left[\ln|x|\right] = \sec^2(\ln|x|)\frac{1}{x}$$

$$= \boxed{\frac{\sec^2(\ln|x|)}{x}}$$

4. Find the equation of the tangent line to the graph of $f(x) = 2\ln(x)$ at the point $(e, f(e))$.

Slope of tangent to $y=f(x)$ at $(x, f(x))$ is $f'(x) = 2\frac{1}{x} = \frac{2}{x}$

slope of tangent to $y=f(x)$ at $(e, f(e))$ is $f'(e) = \frac{2}{e} = m$

Point on tangent: $(x_0, y_0) = (e, f(e)) = (e, 2\ln(e)) = (e, 2)$

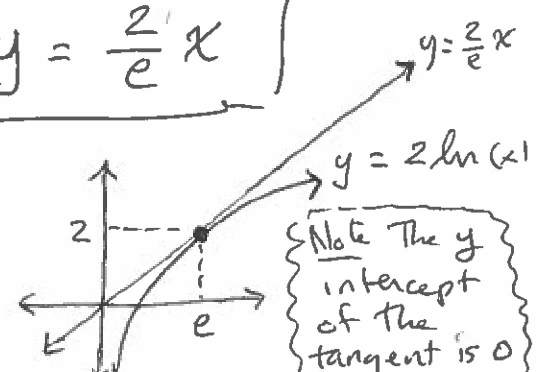
By point-slope formula:

$$y - y_0 = m(x - x_0)$$

$$y - 2 = \frac{2}{e}(x - e)$$

$$y - 2 = \frac{2}{e}x - 2$$

$$\boxed{y = \frac{2}{e}x}$$



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QUIZ 11 ♠

MATH 200
October 6, 2021

1. Find the derivative: $y = \left(x^2 + \ln(x)\right)^5$

$$y' = 5\left(x^2 + \ln(x)\right)^4 D_x\left[x^2 + \ln(x)\right]$$
$$= \boxed{5\left(x^2 + \ln(x)\right)^4\left(2x + \frac{1}{x}\right)}$$

2. Find the derivative: $y = \ln\left(x + \cos(x)\right)$ $\begin{cases} y = \ln(u) \\ u = x + \cos(x) \end{cases}$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{u}\left(1 - \sin(x)\right) = \frac{1}{x + \cos(x)}\left(1 - \sin(x)\right)$$
$$= \frac{1 - \sin(x)}{x + \cos(x)}$$

3. Find the derivative: $y = x + \cos\left(\ln|x|\right)$

$$y' = 1 - \sin(\ln|x|)\, D_x\left[\ln|x|\right] = 1 - \sin(\ln|x|)\frac{1}{x}$$
$$= \boxed{1 - \frac{\sin(\ln|x|)}{x}}$$

4. Find the equation of the tangent line to the graph of $f(x) = \ln(x - 1)$ at the point $(2, f(2))$.

Slope of tangent to $y = f(x)$ at $(x, f(x))$ is $f'(x) = \frac{1}{x-1}$

Slope of tangent to $y = f(x)$ at $(2, f(2))$ is $m = f'(2) = \frac{1}{2-1} = \boxed{1}$

Point on tangent is $(2, f(2)) = (2, \ln(2-1)) = (2, \ln(1))$
$= (2, 0)$
$= (x_0, y_0)$

Point-slope formula:

$$y - y_0 = m(x - x_0)$$
$$y - 0 = 1(x - 2)$$
$$\boxed{y = x - 2}$$