Name: Richard

QUIZ 11 ♡

MATH 200

October 10, 2022

1. If $y = \ln\left|x^5 - x^2 + 3x + 1\right|$, then $\dfrac{dy}{dx} =$ $\boxed{\dfrac{5x^4 - 2x + 3}{x^5 - x^2 + 3x + 1}}$

2. $\dfrac{d}{dx}\Big[\big(\cos(x) + \ln(5x+1)\big)^3\Big] =$ $3\big(\cos(x) + \ln(5x+1)\big)^2 D_x\big[\cos(x) + \ln(5x+1)\big]$

$= \boxed{3\big(\cos(x) + \ln(5x+1)\big)^2\left(-\sin(x) + \dfrac{5}{5x+1}\right)}$

3. $D_u\Big[e^{u+\ln|\sin(u)|}\Big] =$ $e^{u+\ln|\sin(u)|} D_u\big[u + \ln|\sin(u)|\big]$

$= e^{u+\ln|\sin(u)|}\left(1 + \dfrac{\cos(u)}{\sin(u)}\right) = \boxed{e^{u+\ln|\sin(u)|}\big(1+\cot(u)\big)}$

4. Find all x for which the tangent line to $f(x) = x\ln(x) - 5x$ is horizontal at $(x, f(x))$.

Solve $f'(x) = 0$ ← 1 pt

$\ln(x) + x\frac{1}{\cancel{x}} - 5 = 0$ ← 1 pt

$\ln(x) + 1 - 5 = 0$

$\ln(x) = 4$

$e^{\ln(x)} = e^4$

$\boxed{x = e^4}$

3 pt.

Name: Richard

QUIZ 11 ♣

MATH 200
October 10, 2022

1. If $y = \ln\left|\sqrt{x}+x\right|$, then $\dfrac{dy}{dx} =$ $\dfrac{D_x\left[\sqrt{x}+x\right]}{\sqrt{x}+x} = \dfrac{D_x\left[x^{1/2}+x\right]}{\sqrt{x}+x}$

$$= \frac{\frac{1}{2}x^{-1/2}+1}{\sqrt{x}+x} = \boxed{\frac{\frac{1}{2\sqrt{x}}+1}{\sqrt{x}+x}}.$$

2. $\dfrac{d}{dw}\Big[\left(e^w+\ln(w)\right)^5\Big] =$

$$= 5\left(e^w+\ln(w)\right)^4 D_x\left[e^w+\ln(w)\right]$$

$$= \boxed{5\left(e^w+\ln(w)\right)^4\left(e^w+\tfrac{1}{w}\right)}$$

3. $D_x\Big[x^4\ln\left|x^3+x^2+x\right|\Big] =$

$$= 4x^3\ln\left|x^3+x^2+x\right| + x^4\,\frac{3x^2+2x+1}{x^3+x^2+x}$$

$$= 4x^3\ln\left|x^3+x^2+x\right| + x^4\,\frac{3x^2+2x+1}{x(x^2+x+1)}$$

$$= \boxed{4x^3\ln\left|x^3+x^2+x\right| + \frac{3x^5+2x^4+x^3}{x^2+x+1}}$$

4. Find all x for which the tangent line to $f(x) = 3x + x\ln(x)$ is horizontal at $(x, f(x))$.

Solve $f'(x) = 0$ ← 1 pt

$3 + \ln(x) + x\frac{1}{x} = 0$ ← 1 pt

$$\left.\begin{aligned} 3 + \ln(x) + 1 &= 0 \\ \ln(x) &= -4 \\ e^{\ln(x)} &= e^{-4} \\ \boxed{x = e^{-4}} &\boxed{= \frac{1}{e^4}} \end{aligned}\right\} \text{3 pt.}$$

Name: Richard

QUIZ 11 ◊

MATH 200
October 10, 2022

1. If $u = \ln|4e^w - w|$, then $\dfrac{du}{dw} =$ $\boxed{\dfrac{4e^{w} - 1}{4e^{w} - w}}$

2. $\dfrac{d}{dx}\left[(\ln(x)+x)^2\right] = 2\left(\ln(x) + x\right)^{2-1}\left(\frac{1}{x} + 1\right) = \boxed{2(\ln(x)+x)\left(\frac{1}{x}+1\right)}$

3. $D_x\left[\dfrac{1+\ln|x|}{1-\ln|x|}\right] = \dfrac{\frac{1}{x}(1-\ln|x|) - (1+\ln|x|)\left(-\frac{1}{x}\right)}{(1-\ln|x|)^2}$

$$= \frac{\frac{1}{x} - \frac{\ln|x|}{x} + \frac{1}{x} + \frac{\ln|x|}{x}}{(1-\ln|x|)^2} = \frac{\frac{2}{x}}{(1-\ln|x|)^2} = \boxed{\frac{2}{x(1-\ln|x|)^2}}$$

4. Find all x for which the tangent line to $f(x) = x + \ln(x^2+1)$ is horizontal at $(x, f(x))$.

Solve $f'(x) = 0$

$$1 + \frac{2x}{x^2+1} = 0$$

$$(x^2+1)\left(1 + \frac{2x}{x^2+1}\right) = 0\,(x^2+1)$$

$$x^2 + 1 + 2x = 0$$

$$x^2 + 2x + 1 = 0$$

$$\longrightarrow (x+1)(x+1) = 0$$

$$\Downarrow$$

$$\boxed{x = -1}$$

Name: Richard

QUIZ 11 ♠

MATH 200

October 10, 2022

1. If $y = \ln|x^3 + \tan(x)|$, then $\dfrac{dy}{dx} =$ $\boxed{\dfrac{3x^2 + \sec^2(x)}{x^3 + \tan(x)}}$

2. $\dfrac{d}{dx}\Big[\left(\ln|x + \sin(x)|\right)^2 \Big] =$ $2\left(\ln|x+\sin(x)|\right)^1 D_x\left[\ln|x+\sin(x)|\right]$

$= \boxed{2\ln|x+\sin(x)| \cdot \dfrac{1+\cos(x)}{x+\sin(x)}}$

3. $D_w\Big[\cos\left(\ln|w^2e^w|\right)\Big] =$ $-\sin\left(\ln|w^2e^w|\right) D_w\left[\ln|w^2e^w|\right]$

$= -\sin\left(\ln|w^2e^w|\right) \dfrac{2we^w + w^2e^w}{w^2e^w}$

$= -\sin\left(\ln|w^2e^w|\right) \dfrac{we^w(2+w)}{w^2e^w} = \boxed{-\sin\left(\ln|w^2e^w|\right) \dfrac{2+w}{w}}$

4. Find all x for which the tangent line to $f(x) = \dfrac{x}{2} + \ln\left(2x^2 + 8\right)$ is horizontal at $(x, f(x))$.

Solve $f'(x) = 0$

$\dfrac{1}{2} + \dfrac{4x}{2x^2+8} = 0$

$(2x^2+8)\left(\dfrac{1}{2} + \dfrac{4x}{2x^2+8}\right) = 0\,(2x^2+8)$

$x^2 + 4 + 4x = 0$

$x^2 + 4x + 4 = 0$

$\longrightarrow (x+2)(x+2) = 0$

$\Downarrow$

$\boxed{x = -2}$