Name: Richard

QUIZ 12 ♡

MATH 200
October 11, 2021

1. Find the derivative: $y = \cos(\pi x) \ln|5x|$ ← (Product rule)

$$y' = -\sin(\pi x)\pi \cdot \ln|5x| + \cos(\pi x)\frac{1}{5x}\cdot 5$$

$$= \boxed{-\pi\sin(\pi x)\ln|5x| + \frac{\cos(\pi x)}{x}}$$

2. Find the derivative: $y = \sin^{-1}(x^5+1)$

$$y' = \frac{1}{\sqrt{1-(x^5+1)^2}} D_x\left[x^5+1\right] = \frac{1}{\sqrt{1-(x^5+1)^2}}5x^4 = \boxed{\frac{5x^4}{\sqrt{1-(x^5+1)^2}}}$$

3. Find the derivative: $y = \left(1+\tan^{-1}(x)\right)^5$

$$y' = 5\left(1+\tan^{-1}(x)\right)^4 D_x\left[1+\tan^{-1}(x)\right]$$

$$= 5\left(1+\tan^{-1}(x)\right)^4\left(0+\frac{1}{1+x^2}\right) = \boxed{\frac{5\left(1+\tan^{-1}(x)\right)^4}{1+x^2}}$$

4. A rocket, moving straight up after launch, has a height of $s(t) = t^2 - 8t + 91$ meters at time t (seconds). Find the rocket's velocity when it is 100 meters high. (Assume $t \geq 0$.)

First lets find when the rocket is 100 m. high.
For this we need to solve $s(t) = 100$

$$t^2 - 8t + 91 = 100$$
$$t^2 - 8t - 9 = 0$$
$$(t+1)(t-9) = 0$$
$$t = -1 \qquad t = 9$$

So rocket is 100 m. high at time $t = 9$ sec.
So we need to plug $t = 9$ into formula for velocity.

$V(t) = s'(t) = 2t - 8$ ⟹ Ans $V(9) = 2\cdot 9 - 8 = \boxed{10\ m/sec}$

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QUIZ 12 ◊

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1. Find the derivative: $y = \sec^{-1}(3x)$

$$y' = \frac{1}{|3x|\sqrt{(3x)^2-1}} D_x[3x] = \boxed{\frac{3}{|3x|\sqrt{9x^2-1}}}$$

2. Find the derivative: $y = 3x\tan^{-1}(x)$ ← (product rule)

$$y' = 3\tan^{-1}(x) + 3x\frac{1}{1+x^2} = \boxed{3\tan^{-1}(x) + \frac{3x}{1+x^2}}$$

3. Find the derivative: $y = \ln|\sin^{-1}(x)|$

$$y' = \frac{1}{\sin^{-1}(x)} D_x\left[\sin^{-1}(x)\right] = \frac{1}{\sin^{-1}(x)}\frac{1}{\sqrt{1-x^2}} = \boxed{\frac{1}{\sin^{-1}(x)\sqrt{1-x^2}}}$$

4. A rocket, moving straight up after launch, has a height of $s(t) = t^2 - 6t + 100$ meters at time t (seconds). Find the rocket's height when its velocity is 14 meters per second.

Velocity at time t is $v(t) = s'(t) = 2t - 6$ m/sec.
To find the time t at which velocity is 14 m/sec we need to solve the equation $v(t) = 14$

$$2t - 6 = 14$$
$$2t = 20$$
$$t = 10 \text{ sec.}$$

Thus velocity is 14 m/sec at time $t = 10$ seconds.
At this time the height is

$$s(10) = 10^2 - 6\cdot 10 + 100 = 100 - 60 + 100 = \boxed{140 \text{ meters}}$$

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QUIZ 12 ♣

MATH 200
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1. Find the derivative: $y = e^{-x} \ln|3x|$

$$y' = e^{-x}(-1)\ln|3x| + e^{-x}\frac{1}{3x}\cdot 3 = \boxed{\frac{e^{-x}}{x} - e^{-x}\ln|3x|}$$

2. Find the derivative: $y = \sin^{-1}(\ln|x|)$

$$y' = \frac{1}{\sqrt{1-(\ln|x|)^2}} D_x\left[\ln|x|\right] = \boxed{\frac{1}{\sqrt{1-(\ln|x|)^2}\,x}}$$

3. Find the derivative: $y = \ln\left|\sin^{-1}(x)\right|$

$$y' = \frac{1}{\sin^{-1}(x)} D_x\left[\sin^{-1}(x)\right] = \boxed{\frac{1}{\sin^{-1}(x)\sqrt{1-x^2}}}$$

(chain rule)

4. A rocket, moving straight up after launch, has a height of $s(t) = 5t^3 - 10t$ meters at time t (seconds). Find the rocket's velocity when its acceleration is 300 meters per second per second.

Position at time t: $s(t) = 5t^3 - 10t$ (meters)

Velocity at time t: $v(t) = s'(t) = 15t^2 - 10$ (meters/sec)

Acceleration at time t: $a(t) = v'(t) = 30t$ (meters/sec/sec)

To find the time t at which the acceleration is 300 m/s/s we must solve
$$a(t) = 300$$
$$30t = 300$$
$$t = 10 \text{ sec}$$

Thus acceleration is 300 m/s/s at the time $t = 10$ sec.
The velocity at this time is $v(10) = 15\cdot 10^2 - 10 = \boxed{1490 \text{ m/s}}$

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QUIZ 12 ♠

MATH 200

October 11, 2021

1. Find the derivative: $y = \sin^{-1}\left(7\ln(x)\right)$

$$y' = \frac{1}{\sqrt{1-(7\ln(x))^2}} D_x\left[7\ln(x)\right] = \frac{1}{\sqrt{1-(7\ln(x))^2}} \cdot \frac{7}{x}$$

$$= \boxed{\frac{7}{x\sqrt{1-(7\ln(x))^2}}}$$

2. Find the derivative: $y = \ln\left(\tan^{-1}(x)\right)$

$$y' = \frac{1}{\tan^{-1}(x)} D_x\left[\tan^{-1}(x)\right] = \frac{1}{\tan^{-1}(x)}\ \frac{1}{1+x^2} = \boxed{\frac{1}{\tan^{-1}(x)(1+x^2)}}$$

3. Find the derivative: $y = \left(x + \sin^{-1}(x)\right)^8$

$$y' = 8\left(x+\sin^{-1}(x)\right)^7 D_x\left[x+\sin^{-1}(x)\right]$$

$$= \boxed{8\left(x+\sin^{-1}(x)\right)^7\left(1+\frac{1}{\sqrt{1-x^2}}\right)}$$

4. A rocket, moving straight up after launch, has a height of $s(t) = 5t^3 + 10t$ meters at time t (seconds). Find the rocket's height when its acceleration is 60 meters per second per second.

height: $S(t) = 50t^3 + 10t$ (at time t)

velocity: $V(t) = 15t^2 + 10$ (at time t)

acceleration: $a(t) = V'(t) = 30t$ (at time t)

To find when acceleration is 60 m/sec/sec, we need to solve $a(t) = 60$, i.e. $30t = 60$

$t = 2$

Thus acceleration is 60 m/sec/sec when $t = 2$.

Height at this time is $S(2) = 5\cdot 2^3 + 10\cdot 2 = \boxed{60 \text{ meters}}$