Name: Richard

QUIZ 12 ♡

MATH 200
October 17, 2022

1. Find the derivative: $y = \sin^{-1}\left(x^5 - 3x^2\right)$

$$y' = \frac{1}{\sqrt{1-(x^5-3x^2)^2}}(5x^4-6x) = \boxed{\frac{5x^4-6x}{\sqrt{1-(x^5-3x^2)^2}}}$$

$$= \boxed{\frac{5x^4-6x}{\sqrt{1-x^{10}+6x^7-9x^4}}}$$

2. Find the derivative: $y = \left(\tan^{-1}(x)\right)^5$

$$y' = 5\left(\tan^{-1}(x)\right)^4 \frac{1}{1+x^2} = \boxed{\frac{5\left(\tan^{-1}(x)\right)^4}{1+x^2}}$$

3. Find the derivative: $y = \dfrac{\sec^{-1}(x)}{e^x}$

$$y' = \frac{\frac{1}{|x|\sqrt{x^2-1}}e^x - \sec^{-1}(x)e^x}{(e^x)^2}$$

$$= \frac{e^x\left(\frac{1}{|x|\sqrt{x^2-1}} - \sec^{-1}(x)\right)}{e^x e^x} = \boxed{\frac{\frac{1}{|x|\sqrt{x^2-1}} - \sec^{-1}(x)}{e^x}}$$

4. Suppose $f(x)$ is the number of liters of fuel in a rocket when it is x miles above the Earth's surface. Explain in simple terms the meaning of the statement $f'(20) = -8$.

When the rocket is 20 miles high, it is using fuel at a rate of -8 liters per mile. At that rate it would burn 8 liters to go an additional mile high.

Name: Richard

QUIZ 12 ♣

MATH 200
October 17, 2022

1. Find the derivative: $y = \tan^{-1}\left(x^5 - 3x^2\right)$

$$y' = \frac{1}{1 + (x^5 - 3x^2)^2}\left(5x^4 - 6x\right) = \boxed{\frac{5x^4 - 6x}{1 + (x^5 - 3x^2)^2}}$$

$$= \boxed{\frac{5x^4 - 6x}{1 + x^{10} - 6x^7 + 9x^4}}$$

2. Find the derivative: $y = \left(\sin^{-1}(x)\right)^5$

$$y' = 5\left(\sin^{-1}(x)\right)^4 \frac{1}{\sqrt{1 - x^2}} = \boxed{\frac{5\left(\sin^{-1}(x)\right)^4}{\sqrt{1 - x^2}}}$$

3. Find the derivative: $y = \ln(x)\sec^{-1}(x)$

$$y' = \frac{1}{x}\sec^{-1}(x) + \ln(x)\frac{1}{|x|\sqrt{x^2 - 1}}$$

$$= \boxed{\frac{\sec^{-1}(x)}{x} + \frac{\ln(x)}{|x|\sqrt{x^2 - 1}}}$$

4. Suppose $f(x)$ is the number of liters of fuel in a rocket when it is x miles above the Earth's surface. Explain in simple terms the meaning of the statement $f'(20) = -8$.

When the rocket is 20 miles high it is using fuel at a rate of -8 liters per mile. At that rate it would burn 8 liters to go an additional mile high.

Name: Richard

QUIZ 12 ◇

MATH 200
October 17, 2022

1. Find the derivative: $y = \sec^{-1}\left(x^5 - 3x^2\right)$

$$y' = \frac{1}{|x^5-3x^2|\sqrt{(x^5-3x^2)^2-1}} D_x\left[x^5-3x^2\right]$$

$$= \boxed{\frac{5x^4-6x}{|x^5-3x^2|\sqrt{(x^5-3x^2)^2-1}}}$$

2. Find the derivative: $y = \left(\sin^{-1}(x)\right)^5$

$$y' = 5\left(\sin^{-1}(x)\right)^4 D_x\left[\sin^{-1}(x)\right]$$

$$= \boxed{\frac{5\left(\sin^{-1}(x)\right)^4}{\sqrt{1-x^2}}}$$

3. Find the derivative: $y = e^{5x}\tan^{-1}(x)$

$$y' = D_x\left[e^{5x}\right]\tan^{-1}(x) + e^{5x} D_x\left[\tan^{-1}(x)\right]$$

$$= \boxed{5e^{5x}\tan^{-1}(x) + \frac{e^{5x}}{1+x^2}}$$

4. Consider the function $h(x)$, where $h(x)$ equals the elevation (in feet above sea level) x miles due west of your present location. Suppose $h'(75) = 5$. Explain what this means.

At the point 75 miles due west from your present location, elevation is increasing at a rate of 5 feet per mile (so that point is on a slight incline, increasing 5 feet in one mile)
[i.e. go one mile further west, and expect to go up 5 feet.]

Name: Richard

QUIZ 12 ♠

MATH 200
October 17, 2022

1. Find the derivative: $y = \sin^{-1}\left(x^5 - 3x^2\right)$

$$y' = \frac{1}{\sqrt{1-(x^5-3x^2)^2}} D_x\left[x^5 - 3x^2\right] = \boxed{\frac{5x^4-6x}{\sqrt{1-(x^5-3x^2)^2}}}$$

$$= \boxed{\frac{5x^4-6x}{\sqrt{1-x^{10}+6x^7-9x^4}}}$$

2. Find the derivative: $y = 3\left(\tan^{-1}(x)\right)^4$

$$y' = 12\left(\tan^{-1}(x)\right)^3 D_x\left[\tan^{-1}(x)\right] = \boxed{\frac{12\left(\tan^{-1}(x)\right)^3}{1+x^2}}$$

3. Find the derivative: $y = \sec(x)\sec^{-1}(x)$

$$y' = D_x[\sec(x)]\sec^{-1}(x) + \sec(x)\, D_x\left[\sec^{-1}(x)\right]$$

$$= \boxed{\sec(x)\tan(x)\sec^{-1}(x) + \sec(x)\frac{1}{|x|\sqrt{x^2-1}}}$$

4. Consider the function $h(x)$, where $h(x)$ equals the elevation (in feet above sea level) x miles due west of your present location. Suppose $h'(75) = 5$. Explain what this means.

At the point 75 miles due west of your present location elevation is increasing at a rate of 5 feet per mile (i.e. that point is on a slight incline, climbing 5 feet in one mile)

[Go one mile further west, expect to go up 5 feet]