Name: Richard

QUIZ 12

MATH 200
October 3, 2023

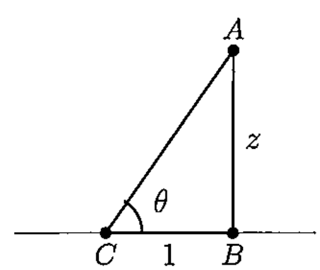
1. $D_x\left[\sec^{-1}(x)\right] =$ $\boxed{\dfrac{1}{|x|\sqrt{x^2-1}}}$

2. $D_x\left[\sin^{-1}\left(x^3+3x\right)\right] =$ $\dfrac{1}{\sqrt{1-(x^3+3x)^2}}(3x^2+3) = \boxed{\dfrac{3x^2+3x}{\sqrt{1-(x^3+3x)^2}}}$

3. $D_x\left[\sqrt{\tan^{-1}(x)}\right] =$ $D_x\left[\left(\tan^{-1}(x)\right)^{\frac{1}{2}}\right] = \frac{1}{2}\left(\tan^{-1}(x)\right)^{\frac{1}{2}-1} D_x\left[\tan^{-1}(x)\right]$

$= \frac{1}{2}\left(\tan^{-1}(x)\right)^{-\frac{1}{2}}\dfrac{1}{1+x^2} = \boxed{\dfrac{1}{2\sqrt{\tan^{-1}(x)}\,(1+x^2)}}$

4. An object (at point A) rises vertically above a point B on the ground. A camera on the ground (at a point C), 1 mile from B, tracks the object and forms an angle θ of inclination, as illustrated. Find the function giving the rate of change of θ with respect to the object's height z (in miles).



$\tan(\theta) = \dfrac{opp}{adj} = \dfrac{z}{1} = z$

Therefore $\theta = \tan^{-1}(z)$

Rate of change of θ is

$\dfrac{d\theta}{dz} = D_z\left[\tan^{-1}(z)\right] = \boxed{\dfrac{1}{1+z^2} \text{ radians/mile}}$

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2. $D_x\left[\sqrt{\sec^{-1}(x)}\right] = D_x\left[\left(\sec^{-1}(x)\right)^{\frac{1}{2}}\right] = \frac{1}{2}\left(\sec^{-1}(x)\right)^{\frac{1}{2}-1} D_x\left[\sec^{-1}(x)\right]$

$$= \frac{1}{2}\left(\sec^{-1}(x)\right)^{-\frac{1}{2}}\frac{1}{|x|\sqrt{x^2-1}} = \boxed{\frac{1}{2\sqrt{\sec^{-1}(x)}\,|x|\sqrt{x^2-1}}}$$

3. $D_x\left[\tan^{-1}\left(x^3+3x\right)\right] = \dfrac{1}{1+\left(x^3+3x\right)^2} D_x\left[x^3+3x\right]$

$$= \boxed{\frac{3x^2+3}{1+x^6+6x^4+9x^2}}$$

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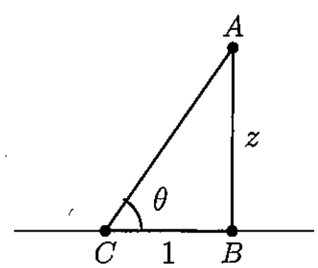

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