Name: Richard

QUIZ 12

MATH 200
October 1, 2024

1. Find all x for which the tangent to the graph of $f(x) = \sin^{-1}(x)$ at $(x, f(x))$ has slope 2.

Solve: $f'(x) = 2$

$$\frac{1}{\sqrt{1-x^2}} = 2$$

$$1 = 2\sqrt{1-x^2}$$

$$1^2 = \left(2\sqrt{1-x^2}\right)^2$$

$$1 = 4(1-x^2)$$

$$1 = 4 - 4x^2$$

$$4x^2 = 3$$

$$x^2 = \frac{3}{4}$$

$$x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$$

Ans $x = \pm\frac{\sqrt{3}}{2}$

2. $D_x\left[\sec^{-1}(x^3)\right] = \dfrac{1}{|x^3|\sqrt{(x^3)^2-1}}(3x^2) = \dfrac{3x^2}{|x^3|\sqrt{x^6-1}} = \boxed{\dfrac{3}{|x|\sqrt{x^6-1}}}$

3. $D_x\left[(\tan^{-1}(x))^3\right] = 3\left(\tan^{-1}(x)\right)^2 D_x\left[\tan^{-1}(x)\right]$

$$= 3\left(\tan^{-1}(x)\right)^2 \frac{1}{1+x^2} = \boxed{\frac{3\left(\tan^{-1}(x)\right)^2}{1+x^2}}$$

4. An object moving on a straight line is $s(t) = t^3 - 3t^2$ feet from its starting point at time t seconds. Find its acceleration when its velocity is -3 feet per second.

Velocity: $v(t) = s'(t) = 3t^2 - 6t$

Acceleration: $a(t) = 6t - 6$

To find when velocity is -3, solve $v(t) = -3$

$$3t^2 - 6t = -3$$

$$3t^2 - 6t + 3 = 0$$

$$3(t^2 - 2t + 1) = 0$$

$$3(t-1)^2 = 0$$

$t = 1$ sec

Velocity is -3 ft/sec at $t = 1$ sec

Acceleration at time $t = 1$ is $a(1) = 6 \cdot 1 - 6 = 0$

Answer: Acceleration is 0 ft/sec^2

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1. Find all x for which the tangent to the graph of $f(x) = \sin^{-1}(x)$ at $(x, f(x))$ has slope 5.

Solve: $f'(x) = 5$

$$\frac{1}{\sqrt{1-x^2}} = 5$$

$$1 = 5\sqrt{1-x^2}$$

$$1^2 = \left(5\sqrt{1-x^2}\right)^2$$

$$1 = 25(1-x^2)$$

$$1 = 25 - 25x^2$$

$$25x^2 = 24$$

$$x^2 = \frac{24}{25}$$

$$x = \pm\sqrt{\frac{24}{25}} = \pm\frac{\sqrt{24}}{5} = \pm\frac{2\sqrt{3}}{5}$$

Ans: $x = \pm\frac{2\sqrt{3}}{5}$

2. $D_x\left[\left(\sec^{-1}(x)\right)^4\right] = 4\left(\sec^{-1}(x)\right)^3 \frac{1}{|x|\sqrt{x^2-1}} = \boxed{\frac{4\left(\sec^{-1}(x)\right)^3}{|x|\sqrt{x^2-1}}}$

3. $D_x\left[\tan^{-1}\left(x^4\right)\right] = \frac{1}{1+(x^4)^2} 4x^3 = \boxed{\frac{4x^3}{1+x^8}}$

4. An object moving on a straight line is $s(t) = 2+t+t^3$ feet from its starting point at time t seconds. Find its velocity when its acceleration is 12 feet per second per second.

$s'(t) = 1 + 3t^2 = v(t)$ (velocity at time t)

$v'(t) = 0 + 6t = a(t)$ (acceleration at time t)

To find when acceleration is 12 ft/sec², solve

$a(t) = 12 \Rightarrow 6t = 12 \Rightarrow \boxed{t = 2 \text{ sec}}$

The velocity at $t = 2$ seconds is Therefore

$v(2) = 1 + 3(2^2) = \boxed{13 \text{ ft/sec}}$ ← Answer