Name: Richard

QUIZ 12

MATH 200
September 30, 2025

1. Find the slope of the tangent line to the graph of $f(x) = \sin^{-1}(x)$ at the point $\big(1/2, f(1/2)\big)$.

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$f'\left(\tfrac{1}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{1}{\sqrt{1-\frac{1}{4}}} = \frac{1}{\sqrt{\frac{3}{4}}} = \frac{1}{\frac{\sqrt{3}}{2}} = \boxed{\frac{2}{\sqrt{3}}} = \boxed{\frac{2\sqrt{3}}{3}}$$

2. $D_x\Big[\sec^{-1}(x+x^2)\Big] =$ (use chain rule)

$$= \frac{1}{|x+x^2|\sqrt{(x+x^2)^2-1}}\, D_x\big[x+x^2\big] = \boxed{\frac{1+2x}{|x+x^2|\sqrt{x^2+2x^3+x^4-1}}}$$

3. $D_x\left[\frac{1}{x} + x^2\tan^{-1}(x)\right] = -\frac{1}{x^2} + 2x\tan^{-1}(x) + x^2\frac{1}{1+x^2}$

$$= \boxed{\frac{x^2}{1+x^2} + 2x\tan^{-1}(x) - \frac{1}{x^2}}$$

4. An ball, thrown straight, up has a height of $s(t) = 5 + 96t - 16t^2$ feet at time t seconds.

(a) Find the function giving the object's velocity at time t.

$$V(t) = s'(t) = \boxed{96 - 32t \ \text{ft/sec}}$$

(b) At what time t does the object reach its maximum height?

When $V(t) = 0$

$$96 - 32t = 0$$

$$-32t = -96$$

$$t = \frac{-96}{32} = 3$$

Answer: $\boxed{\text{Maximum height at time } t = 3 \text{ seconds}}$

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1. Find the slope of the tangent line to the graph of $f(x) = \sec^{-1}(x)$ at the point $(2, f(2))$.

$$f'(x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$f'(2) = \frac{1}{|2|\sqrt{2^2-1}} = \boxed{\frac{1}{2\sqrt{3}}} = \boxed{\frac{\sqrt{3}}{6}}$$

2. $D_x\left[\tan^{-1}\left(x^2e^x\right)\right] =$

$$\frac{1}{1+(x^2e^x)^2} D_x\left[x^2e^x\right] = \boxed{\frac{2xe^x - x^2e^x}{1+x^4e^{2x}}}$$

3. $D_x\left[\frac{1}{x^2} + x\sin^{-1}(x)\right] = -\frac{2}{x^3} + (1)\sin^{-1}(x) + x\frac{1}{\sqrt{1-x^2}}$

$$= \boxed{\frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x) - \frac{2}{x^3}}$$

4. An ball, thrown straight, up has a height of $s(t) = 10 + 128t - 16t^2$ feet at time t seconds.

(a) Find the function giving the object's velocity at time t.

$$V(t) = s'(t) = 128 - 32t$$

(b) At what time t does the object reach its maximum height?

When $V(t) = 0$

$$128 - 32t = 0$$

$$128 = 32t$$

$$\frac{128}{32} = t$$

$$\boxed{t = 4 \text{ sec}}$$

Answer

Bull reaches its maximum height when $t = 4$ seconds