

1. Find the derivative:
- $y = \sin^{-1}(x^5 - 3x^2)$

$$y' = \frac{1}{\sqrt{1 - (x^5 - 3x^2)^2}} (5x^4 - 6x) = \frac{5x^4 - 6x}{\sqrt{1 - (x^5 - 3x^2)^2}}$$

$$= \frac{5x^4 - 6x}{\sqrt{1 - x^{10} + 6x^7 - 9x^4}}$$

2. Find the derivative:
- $y = (\tan^{-1}(x))^5$

$$y' = 5(\tan^{-1}(x))^4 \frac{1}{1+x^2} = \frac{5(\tan^{-1}(x))^4}{1+x^2}$$

3. Find the derivative:
- $y = \frac{\sec^{-1}(x)}{e^x}$

$$y' = \frac{\frac{1}{x\sqrt{x^2-1}} e^x - \sec^{-1}(x) e^x}{(e^x)^2}$$

$$= \frac{e^x \left(\frac{1}{x\sqrt{x^2-1}} - \sec^{-1}(x) \right)}{e^x e^x} = \frac{\frac{1}{x\sqrt{x^2-1}} - \sec^{-1}(x)}{e^x}$$

4. Suppose
- $f(x)$
- is the number of liters of fuel in a rocket when it is
- x
- miles above the Earth's surface. Explain in simple terms the meaning of the statement
- $f'(20) = -8$
- .

When the rocket is 20 miles high, it is using fuel at a rate of -8 liters per mile. At that rate it would burn 8 liters to go an additional mile high.

1. Find the derivative: $y = \tan^{-1}(x^5 - 3x^2)$

$$y' = \frac{1}{1 + (x^5 - 3x^2)^2} (5x^4 - 6x) = \frac{5x^4 - 6x}{1 + (x^5 - 3x^2)^2}$$

$$= \frac{5x^4 - 6x}{1 + x^{10} - 6x^7 + 9x^4}$$

2. Find the derivative: $y = (\sin^{-1}(x))^5$

$$y' = 5(\sin^{-1}(x))^4 \frac{1}{\sqrt{1-x^2}} = \frac{5(\sin^{-1}(x))^4}{\sqrt{1-x^2}}$$

3. Find the derivative: $y = \ln(x) \sec^{-1}(x)$

$$y' = \frac{1}{x} \sec^{-1}(x) + \ln(x) \frac{1}{|x| \sqrt{x^2 - 1}}$$

$$= \frac{\sec^{-1}(x)}{x} + \frac{\ln(x)}{|x| \sqrt{x^2 - 1}}$$

4. Suppose $f(x)$ is the number of liters of fuel in a rocket when it is x miles above the Earth's surface. Explain in simple terms the meaning of the statement $f'(20) = -8$.

When the rocket is 20 miles high it is using fuel at a rate of -8 liters per mile. At that rate it would burn 8 liters to go an additional mile high.

1. Find the derivative: $y = \sec^{-1}(x^5 - 3x^2)$

$$y' = \frac{1}{|x^5 - 3x^2| \sqrt{(x^5 - 3x^2)^2 - 1}} D_x [x^5 - 3x^2]$$

$$= \frac{5x^4 - 6x}{|x^5 - 3x^2| \sqrt{(x^5 - 3x^2)^2 - 1}}$$

2. Find the derivative: $y = (\sin^{-1}(x))^5$

$$y' = 5 (\sin^{-1}(x))^4 D_x [\sin^{-1}(x)]$$

$$= \frac{5 (\sin^{-1}(x))^4}{\sqrt{1 - x^2}}$$

3. Find the derivative: $y = e^{5x} \tan^{-1}(x)$

$$y' = D_x [e^{5x}] \tan^{-1}(x) + e^{5x} D_x [\tan^{-1}(x)]$$

$$= 5e^{5x} \tan^{-1}(x) + \frac{e^{5x}}{1 + x^2}$$

4. Consider the function $h(x)$, where $h(x)$ equals the elevation (in feet above sea level) x miles due west of your present location. Suppose $h'(75) = 5$. Explain what this means.

At the point 75 miles due west from your present location, elevation is increasing at a rate of 5 feet per mile (so that point is on a slight incline, increasing 5 feet in one mile)
[i.e. go one mile further west, and expect to go up 5 feet.]

1. Find the derivative:
- $y = \sin^{-1}(x^5 - 3x^2)$

$$y' = \frac{1}{\sqrt{1 - (x^5 - 3x^2)^2}} D_x [x^5 - 3x^2] = \frac{5x^4 - 6x}{\sqrt{1 - (x^5 - 3x^2)^2}}$$

$$= \frac{5x^4 - 6x}{\sqrt{1 - x^{10} + 6x^7 - 9x^4}}$$

2. Find the derivative:
- $y = 3(\tan^{-1}(x))^4$

$$y' = 12(\tan^{-1}(x))^3 D_x [\tan^{-1}(x)] = \frac{12(\tan^{-1}(x))^3}{1 + x^2}$$

3. Find the derivative:
- $y = \sec(x) \sec^{-1}(x)$

$$y' = D_x [\sec(x)] \sec^{-1}(x) + \sec(x) D_x [\sec^{-1}(x)]$$

$$= \sec(x) \tan(x) \sec^{-1}(x) + \sec(x) \frac{1}{|x| \sqrt{x^2 - 1}}$$

4. Consider the function
- $h(x)$
- , where
- $h(x)$
- equals the elevation (in feet above sea level)
- x
- miles due west of your present location. Suppose
- $h'(75) = 5$
- . Explain what this means.

At the point 75 miles due west of your present location, elevation is increasing at a rate of 5 feet per mile (i.e. that point is on a slight incline, climbing 5 feet in one mile)
 [Go one mile further west, expect to go up 5 feet]