Name: Richard

QUIZ 13 ♡

MATH 200
October 13, 2021

1. This problem concerns the equation $2\sin(xy) = \sqrt{2}y^2$

(a) Find $\frac{dy}{dx}$.

$$D_x\left[2\sin(xy)\right] = D_x\left[\sqrt{2}y^2\right]$$

$$2\cos(xy)(1\cdot y + xy') = \sqrt{2}\cdot 2\cdot y y'$$

$$2\cos(xy)y + 2\cos(xy)xy' = 2\sqrt{2}yy'$$

$$2\cos(xy)xy' - 2\sqrt{2}yy' = -2\cos(xy)y$$

$$y'\cdot 2(\cos(xy)x - \sqrt{2}y) = -2\cos(xy)y$$

$$y' = \frac{-2y\cos(xy)}{2(x\cos(xy) - \sqrt{2}y)}$$

$$\boxed{\frac{dy}{dx} = \frac{-y\cos(xy)}{x\cos(xy) - \sqrt{2}y}}$$

(b) Use your answer from part (a) to find the slope of the tangent to the graph of $2\sin(xy) = \sqrt{2}y^2$ at the point $(\pi/4, 1)$.

$$\left.\frac{dy}{dx}\right|_{(\frac{\pi}{4},1)} = \frac{-1\cos(\frac{\pi}{4}\cdot 1)}{\frac{\pi}{4}\cos(\frac{\pi}{4}\cdot 1) - \sqrt{2}\cdot 1} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\pi}{4}\frac{\sqrt{2}}{2} - \sqrt{2}}$$

$$= \frac{-\sqrt{2}\left(\frac{1}{2}\right)}{\sqrt{2}\left(\frac{\pi}{8} - 1\right)} = \frac{-\frac{1}{2}}{\frac{\pi}{8} - 1} = \frac{-\frac{1}{2}}{\frac{\pi}{8} - \frac{8}{8}} = \frac{-\frac{1}{2}}{\frac{\pi - 8}{8}} = \boxed{\frac{-4}{\pi - 8}}$$

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QUIZ 13 ♣

MATH 200
October 13, 2021

1. This problem concerns the equation $\ln(xy) = x - y$

(a) Find $\frac{dy}{dx}$.

$$D_x[\ln(xy)] = D_x[x-y]$$

$$\frac{1}{xy}(1\cdot y + xy') = 1 - y'$$

$$1\cdot y + xy' = (1-y')xy$$

$$y + xy' = xy - xyy'$$

$$xy' + xyy' = xy - y$$

$$y'(x + xy) = xy - y$$

$$y' = \frac{xy - y}{x + xy}$$

$$\boxed{\frac{dy}{dx} = \frac{xy - y}{x + xy}}$$

(b) Use your answer from part (a) to find the slope of the tangent to the graph of $\ln(xy) = x - y$ at the point $(1, 1)$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,1)} = \frac{1\cdot 1 - 1}{1 + 1\cdot 1} = \frac{0}{3} = \boxed{0}$$

Name: Richard

QUIZ 13 ◊

MATH 200
October 13, 2021

1. This problem concerns the equation $xy + \cos(xy) = 1$

(a) Find $\frac{dy}{dx}$.

$$D_x\left[xy + \cos(xy)\right] = D_x[1]$$

$$1\cdot y + xy' - \sin(xy)(1\cdot y + xy') = 0$$

$$y + xy' - \sin(xy)y - \sin(xy)xy' = 0$$

$$xy' - \sin(xy)xy' = \sin(xy)y - y$$

$$y'(x - \sin(xy)x) = \sin(xy)y - y$$

$$y' = \frac{\sin(xy)y - y}{x - \sin(xy)x}$$

$$y' = \frac{y(\sin(xy) - 1)}{x(1 - \sin(xy))}$$

$$\boxed{y' = -\frac{y}{x}}$$

$$\boxed{\frac{dy}{dx} = -\frac{y}{x}}$$

(b) Use your answer from part (a) to find the slope of the tangent to the graph of $xy + \cos(xy) = 0$ at the point $(1, 0)$.

$$m = \left.\frac{dy}{dx}\right|_{(x,y)=(1,0)} = -\frac{0}{1} = \boxed{0}$$

Name: Richard

QUIZ 13 ♠

MATH 200

October 13, 2021

1. This problem concerns the equation $x^4 + 2xy + y^4 = \cos(x)$

(a) Find $\frac{dy}{dx}$.

$$D_x\left[x^4 + 2xy + y^4\right] = D_x\left[\cos(x)\right]$$

$$4x^3 + 2y + 2xy' + 4y^3y' = -\sin(x)$$

$$2xy' + 4y^3y' = -\sin(x) - 4x^3 - 2y$$

$$y'(2x + 4y^3) = -\sin(x) - 4x^3 - 2y$$

$$y' = \frac{-\sin(x) - 4x^3 - 2y}{2x + 4y^3}$$

$$\boxed{\frac{dy}{dx} = \frac{-\sin(x) - 4x^3 - 2y}{2x + 4y^3}}$$

(b) Use your answer from part (a) above to find the slope of the tangent to the graph of $x^4 + 2xy + y^4 = \cos(x)$ at the point $(0, 1)$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(0,1)} = \frac{-\sin(0) - 4\cdot 0^3 - 2\cdot 1}{2\cdot 0 + 4\cdot 1^3} = \frac{-2}{4}$$

$$= \boxed{-\frac{1}{2}}$$