Name: Richard

QUIZ 13 ♡

MATH 200
October 19, 2022

1. This problem concerns the equation $\sin(x+y) = x^2$.

(a) Which of the following points is on the graph this equation? $(\pi, 0)$, $(0, \pi)$, (π, π)

Check $(\pi, 0)$: $\sin(\pi+0) \stackrel{?}{=} \pi^2 \Rightarrow 0 \stackrel{?}{=} \pi^2$ NO!

Check $(0, \pi)$: $\sin(0+\pi) \stackrel{?}{=} 0^2 \Rightarrow 0 \stackrel{?}{=} 0$ YES!

Check (π, π): $\sin(\pi+\pi) \stackrel{?}{=} \pi^2 \Rightarrow 0 = \pi^2$ NO!

Conclusion Only the point $(0, \pi)$ is on the graph

(b) Find y'.

$\{y = f(x)\}$

$$D_x[\sin(x+y)] = D_x[x^2]$$

$$\cos(x+y)(1+y') = 2x$$

$$\cos(x+y) + y'\cos(x+y) = 2x$$

$$y'\cos(x+y) = 2x - \cos(x+y)$$

$$\boxed{y' = \frac{2x - \cos(x+y)}{\cos(x+y)}}$$

(c) For each point (x_0, y_0) from part (a) that is on the graph of $\sin(x+y) = x^2$, find the slope of the tangent line to the graph at that point.

$$y'\Big|_{(0,\pi)} = \frac{2 \cdot 0 - \cos(0+\pi)}{\cos(0+\pi)} = \frac{-(-1)}{-1} = \boxed{-1}$$

Name: Richard QUIZ 13 ♣ MATH 200

October 19, 2022

1. This problem concerns the equation $e^{xy} - y^2 = x$.

(a) Which of the following points is on the graph this equation? $(1,0)$, $(-1,0)$, $(1,1)$

Check $(1,0)$: $e^{1\cdot 0} - 0^2 \overset{?}{=} 1 \Rightarrow 1 - 0 \overset{?}{=} 1 \Rightarrow 1 = 1$ $\boxed{\text{YES!}}$

Check $(-1,0)$: $e^{(-1)\cdot 0} - 0^2 \overset{?}{=} -1 \Rightarrow 1 - 0 \overset{?}{=} -1 \Rightarrow 1 = -1$ $\boxed{\text{NO!}}$

Check $(1,1)$: $e^{1\cdot 1} - 1^2 \overset{?}{=} 1 \Rightarrow e - 1 \overset{?}{=} 1$ $\boxed{\text{NO!}}$

Conclusion: $\boxed{\text{Only the point } (1,0) \text{ is on the graph}}$

(b) Find y'.

$y = f(x)$

$$D_x\left[e^{xy} - y^2\right] = D_x[x]$$

$$e^{xy}(1\cdot y + xy') - 2yy' = 1$$

$$ye^{xy} + xe^{xy}y' - 2yy' = 1$$

$$xe^{xy}y' - 2yy' = 1 - ye^{xy}$$

$$y'(xe^{xy} - 2y) = 1 - ye^{xy}$$

$$\boxed{y' = \frac{1 - ye^{xy}}{xe^{xy} - 2y}}$$

(c) For each point (x_0, y_0) from part (a) that is on the graph of $e^{xy} - y^2 = x$, find the slope of the tangent line to the graph at that point.

$$y'\Big|_{(1,0)} = \frac{1 - 0e^{1\cdot 0}}{1\cdot e^{1\cdot 0} - 2\cdot 0} = \frac{1}{1} = \boxed{1}$$

Name: Richard

QUIZ 13 ◊

MATH 200
October 19, 2022

1. This problem concerns the equation $\cos(x+y) = y^2$.

(a) Which of the following points is on the graph this equation? (π, π), $(0,0)$, $(\pi/2, 0)$

Check (π, π): $\cos(\pi+\pi) \stackrel{?}{=} \pi^2 \Rightarrow \cos(2\pi) = \pi^2 \Rightarrow 1 = \pi^2$ $\boxed{\text{NO}}$

Check $(0,0)$: $\cos(0+0) \stackrel{?}{=} 0^2 \Rightarrow \cos(0) \stackrel{?}{=} 0 \Rightarrow 1 = 0$ $\boxed{\text{NO}}$

Check $(\pi/2, 0)$: $\cos(\pi/2 + 0) \stackrel{?}{=} 0^2 \Rightarrow \cos(\frac{\pi}{2}) \stackrel{?}{=} 0 \Rightarrow 0 = 0$ $\boxed{\text{YES}}$

(b) Find y'.

$$D_x[\cos(x+y)] = D_x[y^2]$$

$$-\sin(x+y)(1+y') = 2yy'$$

$$-\sin(x+y) - y'\sin(x+y) = 2yy'$$

$$-y'\sin(x+y) - 2yy' = \sin(x+y)$$

$$y'(-\sin(x+y) - 2y) = \sin(x+y)$$

$$\boxed{y' = \frac{\sin(x+y)}{-\sin(x+y) - 2y}}$$

(c) For each point (x_0, y_0) from part (a) that is on the graph of $\cos(x+y) = y^2$, find the slope of the tangent line to the graph at that point.

$$y'\Big|_{(x,y)=(\frac{\pi}{2}, 0)} = \frac{\sin(\pi/2 + 0)}{-\sin(\pi/2 + 0) - 2\cdot 0} = \frac{\sin(\pi/2)}{-\sin(\pi/2)} = \frac{1}{-1} = \boxed{-1}$$

Name: Richard

QUIZ 13 ♠

MATH 200
October 19, 2022

1. This problem concerns the equation $e^{xy} = y^3 + x^2$.

(a) Which of the following points is on the graph this equation? $\boxed{(1,0),}$ $(0,-1)$, $(1,1)$

Check $(1,0)$: $e^{1\cdot 0} \overset{?}{=} 0^3 + 1^2 \Rightarrow e^0 \overset{?}{=} 1 \Rightarrow 1 = 1$ $\boxed{\text{YES!}}$

Check $(0,-1)$: $e^{0(-1)} \overset{?}{=} (-1)^3 + 0^2 \Rightarrow e^0 \overset{?}{=} -1 \Rightarrow 1 = -1$ $\boxed{\text{NO!}}$

Check $(1,1)$: $e^{1\cdot 1} \overset{?}{=} 1^3 + 1^2 \Rightarrow e \overset{?}{=} 2$ $\boxed{\text{NO!}}$

$\boxed{\text{Conclusion Only the point } (1,0) \text{ is on the graph}}$

(b) Find y'.

$$D_x\left[e^{xy}\right] = D_x\left[y^3 + x^2\right]$$

$$e^{xy}(1\cdot y + xy') = 3y^2y' + 2x$$

$$ye^{xy} + xe^{xy}y' = 3y^2y' + 2x$$

$$xe^{xy}y' - 3y^2y' = 2x - ye^{xy}$$

$$y'(xe^{xy} - 3y^2) = 2x - ye^{xy}$$

$$\boxed{y' = \frac{2x - ye^{xy}}{xe^{xy} - 3y^2}}$$

(c) For each point (x_0, y_0) from part (a) that is on the graph of $e^{xy} = y^3 + x^2$, find the slope of the tangent line to the graph at that point.

$$y'\Big|_{(x,y)=(1,0)} = \frac{2\cdot 1 - 0e^{1\cdot 0}}{1e^{1\cdot 0} - 3\cdot 0^2} = \frac{2}{1} = \boxed{2}$$