Name: Richard

QUIZ 13

MATH 200
October 3, 2024

1. This problem concerns the equation $e^{xy} = e^{2x}$.

(a) Which of the following points is on the graph this equation? $(1,2)$, $(2,1)$, $(1/2,0)$

Test $(1,2)$: $e^{1\cdot 2} \stackrel{?}{=} e^{2\cdot 1} \Rightarrow e^2 = e^2$ (Yes!)

Test $(2,1)$: $e^{2\cdot 1} \stackrel{?}{=} e^{2\cdot 2} \Rightarrow e^2 = e^4$ (No!)

Test $(\frac{1}{2},0)$: $e^{\frac{1}{2}\cdot 0} \stackrel{?}{=} e^{2\cdot\frac{1}{2}} \Rightarrow e^0 = e^1$ (No!)

Answer: Only the point $(1,2)$ is on the graph

(b) Find y'.

$$D_x[e^{xy}] = D_x[e^{2x}]$$

$$e^{xy}(1\cdot y + xy') = e^{2x}\cdot 2$$

$$ye^{xy} + xy'e^{xy} = 2e^{2x}$$

$$xy'e^{xy} = 2e^{2x} - ye^{xy}$$

$$y' = \frac{2e^{2x} - ye^{xy}}{xe^{xy}}$$

(c) For each point (x_0, y_0) from part (a) that is on the graph of $e^{xy} = e^{2x}$, find the slope of the tangent line to the graph at that point.

$$y'\Big|_{(x,y)=(1,2)} = \frac{2e^{2\cdot 1} - 2e^{2\cdot 1}}{1\cdot e^{2\cdot 1}} = \frac{0}{e^2} = 0$$

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1. This problem concerns the equation $\sin(xy) = \cos(x/2)$.

(a) Which of the following points is on the graph this equation? $(2\pi, 3)$, $(\pi, 2)$, $(0, \pi)$

Test $(\pi, 3)$: $\sin(2\pi \cdot 3) \stackrel{?}{=} \cos\left(\frac{2\pi}{2}\right) \Rightarrow 0 = -1$ NO!

Test $(\pi, 2)$: $\sin(\pi \cdot 2) \stackrel{?}{=} \cos\left(\frac{\pi}{2}\right) \Rightarrow 0 = 0$ YES!

Test $(0, \pi)$: $\sin(0 \cdot \pi) \stackrel{?}{=} \cos\left(\frac{0}{2}\right) \Rightarrow 0 = 1$ NO!

Answer: Only the point $(\pi, 2)$ is on the graph

(b) Find y'.

$$D_x\left[\sin(xy)\right] = D_x\left[\cos\left(\frac{x}{2}\right)\right]$$

$$\cos(xy)(1 \cdot y + x y') = -\sin\left(\frac{x}{2}\right)\frac{1}{2}$$

$$y\cos(xy) + xy'\cos(xy) = -\frac{1}{2}\sin\left(\frac{x}{2}\right)$$

$$xy'\cos(xy) = -\frac{1}{2}\sin\left(\frac{x}{2}\right) - y\cos(xy)$$

$$y' = \frac{-\frac{1}{2}\sin\left(\frac{x}{2}\right) - y\cos(xy)}{x\cos(xy)}$$

(c) For each point (x_0, y_0) from part (a) that is on the graph of $\sin(xy) = \cos(x/2)$, find the slope of the tangent line to the graph at that point.

$$y'\Big|_{(x,y)=(\pi,2)} = \frac{-\frac{1}{2}\sin\left(\frac{\pi}{2}\right) - 2\cos(2\pi)}{\pi\cos(2\pi)} = \frac{-\frac{1}{2} - 2(1)}{\pi \cdot 1} = \boxed{-\frac{5}{2\pi}}$$