

1. Use logarithmic differentiation to find the derivative of the function $y = (x^4 + x)^{5x}$.

$$y = (x^4 + x)^{5x}$$

$$\ln |y| = \ln |(x^4 + x)^{5x}|$$

$$\ln |y| = 5x \ln |x^4 + x|$$

$$D_x [\ln |y|] = D_x [5x \cdot \ln |x^4 + x|]$$

$$\frac{y'}{y} = 5 \ln |x^4 + x| + 5x \frac{4x^3 + 1}{x^4 + x}$$

$$y' = y \left(5 \ln |x^4 + x| + \frac{20x^4 + 5x}{x^4 + x} \right)$$

$$y' = (x^4 + x)^{5x} \left(5 \ln |x^4 + x| + \frac{20x^4 + 5x}{x^4 + x} \right)$$

1. Use logarithmic differentiation to find the derivative of the function $y = x^{x^2+5}$.

$$y = x^{x^2+5}$$

$$\ln|y| = \ln|x^{x^2+5}|$$

$$\ln|y| = (x^2+5) \ln|x|$$

$$D_x [\ln|y|] = D_x [(x^2+5) \ln|x|]$$

$$\frac{y'}{y} = (2x) \ln|x| + (x^2+5) \frac{1}{x}$$

$$y' = y \left(2x \ln|x| + \frac{x^2+5}{x} \right)$$

$$y' = x^{x^2+5} \left(2x \ln|x| + \frac{x^2+5}{x} \right)$$

1. Use logarithmic differentiation to find the derivative of the function $y = (x^2 + 1)^x$.

$$y = (x^2 + 1)^x$$

$$\ln(y) = \ln((x^2 + 1)^x)$$

$$\ln(y) = x \cdot \ln(x^2 + 1)$$

$$D_x[\ln(y)] = D_x[x \ln(x^2 + 1)]$$

$$\frac{y'}{y} = 1 \cdot \ln(x^2 + 1) + x \frac{2x + 0}{x^2 + 1}$$

$$y' = y \left(\ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right)$$

$$y' = (x^2 + 1)^x \left(\ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right)$$

1. Use logarithmic differentiation to find the derivative of the function $y = (x^2 + 1)^{x+1}$.

$$y = (x^2 + 1)^{x+1}$$

$$\ln|y| = \ln|(x^2 + 1)^{x+1}|$$

$$\ln|y| = (x+1) \cdot \ln|x^2 + 1|$$

$$D_x [\ln|y|] = D_x [(x+1) \cdot \ln|x^2 + 1|]$$

$$\frac{y'}{y} = 1 \cdot \ln|x^2 + 1| + (x+1) \cdot \frac{2x}{x^2 + 1}$$

$$y' = y \left(\ln|x^2 + 1| + \frac{2x^2 + 2x}{x^2 + 1} \right)$$

$$y' = (x^2 + 1)^{x+1} \left(\ln|x^2 + 1| + \frac{2x^2 + 2x}{x^2 + 1} \right)$$