

1. Use logarithmic differentiation to find the derivative of $y = \frac{x^8 \sqrt[3]{\sin(x)}}{\cos(x)^x} = \frac{x^8 (\sin(x))^{1/3}}{\cos(x)^x}$

$$\ln|y| = \ln \left| \frac{x^8 (\sin(x))^{1/3}}{\cos(x)^x} \right|$$

$$\ln|y| = \ln|x^8| + \ln|(\sin(x))^{1/3}| - \ln|\cos(x)^x|$$

$$\ln|y| = 8\ln|x| + \frac{1}{3}\ln|\sin(x)| - x\ln|\cos(x)|$$

$$D_x[\ln|y|] = D_x \left[8\ln|x| + \frac{1}{3}\ln|\sin(x)| - x\ln|\cos(x)| \right]$$

$$\frac{y'}{y} = \frac{8}{x} + \frac{1}{3} \frac{\cos(x)}{\sin(x)} - 1 \ln|\cos(x)| - x \frac{(-\sin(x))}{\cos(x)}$$

$$y' = y \left(\frac{8}{x} + \frac{1}{3} \cot(x) - \ln|\cos(x)| + x \tan(x) \right)$$

$$y' = \frac{x^8 \sqrt[3]{\sin(x)}}{\cos(x)^x} \left(\frac{8}{x} + \frac{1}{3} \cot(x) - \ln|\cos(x)| + x \tan(x) \right)$$

Name: Richard

QUIZ 14

MATH 200
October 8, 2024

1. Use logarithmic differentiation to find the derivative of $y = \frac{\cos(x)^x}{x^8 \sqrt[3]{\sin(x)}} = \frac{\cos(x)^x}{x^8 (\sin(x))^{1/3}}$

$$\ln|y| = \ln \left| \frac{\cos(x)^x}{x^8 (\sin(x))^{1/3}} \right|$$

$$\ln|y| = \ln|\cos(x)^x| - \ln|x^8(\sin(x))^{1/3}|$$

$$\ln|y| = x \ln|\cos(x)| - (\ln|x^8| + \ln|(\sin(x))^{1/3}|)$$

$$\ln|y| = x \ln|\cos(x)| - 8 \ln|x| - \frac{1}{3} \ln|\sin(x)|$$

$$D_x[\ln|y|] = D_x \left[x \ln|\cos(x)| - 8 \ln|x| - \frac{1}{3} \ln|\sin(x)| \right]$$

$$\frac{y'}{y} = 1 \cdot \ln|\cos(x)| + x \frac{-\sin(x)}{\cos(x)} - 8 \frac{1}{x} - \frac{1}{3} \frac{\cos(x)}{\sin(x)}$$

$$y' = y \left(\ln|\cos(x)| - x \tan(x) - \frac{8}{x} - \frac{1}{3} \cot(x) \right)$$

$$y' = \frac{\cos(x)^x}{x^8 \sqrt[3]{\sin(x)}} \left(\ln|\cos(x)| - x \tan(x) - \frac{8}{x} - \frac{1}{3} \cot(x) \right)$$