Name: Richard

QUIZ 14

MATH 200
October 7, 2025

1. this problem concerns the function $f(x) = (x-1)^{(x-1)}$.

(a) Use logarithmic differentiation to find its derivative.

$$y = (x-1)^{x-1}$$
$$\ln(y) = \ln\left((x-1)^{x-1}\right)$$
$$\ln(y) = (x-1)\ln(x-1)$$
$$D_x[\ln(y)] = D_x[(x-1)\ln(x-1)]$$
$$\frac{y'}{y} = 1\cdot\ln(x-1) + (x-1)\frac{1}{x-1}$$
$$y' = y(\ln(x-1)+1)$$
$$y' = (x-1)^{x-1}(\ln(x-1)+1)$$
$$\boxed{f'(x) = (x-1)^{x-1}(\ln(x-1)+1)}$$

(b) $f'(2) = (2-1)^{2-1}(\ln(2-1)+1) = 1^1(\ln(1)+1) = 1\cdot(0+1) = \boxed{1}$

(c) $f(2) = (2-1)^{2-1} = \boxed{1}$

(d) Find the equation of the tangent line to the graph of $y = f(x)$ at the point $(2, f(2))$.

$= (2,1)$

$$y - y_0 = m(x - x_0)$$
$$y - 1 = 1(x-2)$$
$$\boxed{y = x-1}$$

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1. this problem concerns the function $f(x) = (2x-3)^x$.

(a) Use logarithmic differentiation to find its derivative.

$$y = (2x-3)^x$$
$$\ln(y) = \ln((2x-3)^x)$$
$$\ln(y) = x\ln(2x-3)$$
$$D_x[\ln(y)] = D_x[x\ln(2x-3)]$$
$$\frac{y'}{y} = 1\cdot\ln(2x-3) + x\frac{2}{2x-3}$$
$$y' = y\left(\ln(2x-3) + \frac{2x}{2x-3}\right)$$
$$\boxed{y' = (2x-3)^x\left(\ln(2x-3) + \frac{2x}{2x-3}\right)}$$

(b) $f'(2) = (2\cdot2-3)^2\left(\ln(2\cdot2-3) + \frac{2\cdot2}{2\cdot2-3}\right) = 1^2\left(\ln(1) + \frac{4}{1}\right)$

$= 1\cdot(0+4) = \boxed{4}$

(c) $f(2) = (2\cdot2-3)^2 = 1^2 = \boxed{1}$

(d) Find the equation of the tangent line to the graph of $y = f(x)$ at the point $(2, f(2))$.

$$y - y_0 = m(x - x_0)$$
$$y - 1 = 4(x-2)$$
$$\boxed{y = 4x - 7}$$