Name: Richard

QUIZ 15 ♡

MATH 200
November 1, 2021

1. (10 points) This problem concerns the function $f(x) = x^3 + 3x^2 + 10$.

   (a) Find the intervals on which f increases and on which it decreases.

   $$f'(x) = 3x^2 + 6x = 3x(x+2) = 0$$

   $x=0$ $\quad$ $x=2$

   The critical points are $x=0$ and $x=-2$.

	-2		0		
$- - - - -$		$- - - - -$		$+ + + + + +$	$3x$
$- - - - -$		$+ + + + +$		$+ + + + + +$	$(x+2)$
$+ + + + +$		$- - - - -$		$+ + + + +$	$f'(x) = 3x(x+2)$

   $\boxed{f(x) \text{ increases on } (-\infty,-2) \text{ \& } (0,\infty)}$ $\boxed{f(x) \text{ decreases on } (-2,0)}$

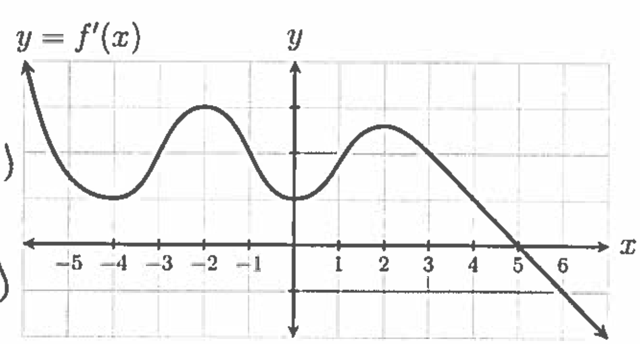
   (b) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

   By first derivative test:

   f has a $\boxed{\text{local maximum at } x=-2}$

   and a $\boxed{\text{local minimum at } x=0}$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.



   (a) State the critical points of f.

   $\boxed{x=5}$ (because $f'(5)=0$)

   (b) State the interval(s) on which f increases.

   $\boxed{(-\infty, 5)}$ (because $f'(x)>0$ there)

   (c) State the interval(s) on which f decreases.

   $\boxed{(5,\infty)}$ (because $f'(x)<0$ there)

   (d) Does f have a local maximum? Where?.

   $\boxed{\text{Yes, at } x=5}$ (because $f'(x)$ changes from + to − at 5)

   (e) Does f have a local minimum? Where?.

   $\boxed{\text{No}}$ (derivative never switches from − to +)

Name: Richard QUIZ 15 ♣ MATH 200

November 1, 2021

1. (10 points) This problem concerns the function $f(x) = x^2e^x + 2$.

(a) Find the intervals on which f increases and on which it decreases.

$$f'(x) = 2xe^x + x^2e^x = xe^x(2+x)$$

$x = 0$ $\qquad$ $x = -2$

Critical points are $x = 0$ and $x = -2$

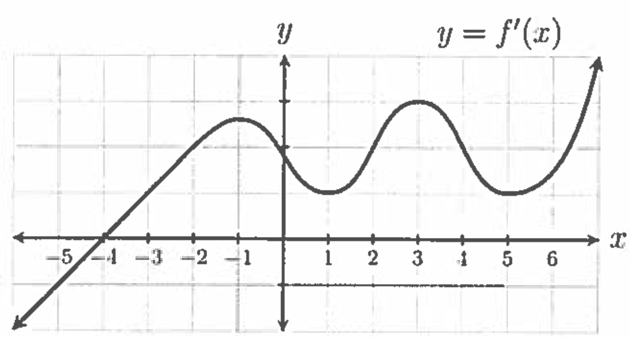
	-2		0		
$- - - - - -$		$- - - -$		$+ + + + + +$	xe^x
$- - - - -$		$+ + + +$		$+ + + + + +$	$(2+x)$
$+ + + + + +$		$- - - -$		$+ + + + +$	$f'(x) = xe^x(2+x)$

$\boxed{f \text{ increases on } (-\infty,-2) \text{ and } (0,\infty)}$ $\boxed{f \text{ decreases on } (-2,0)}$

(b) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

By first derivative test, there is a $\boxed{\text{local max at } x=-2}$ and a $\boxed{\text{local min at } x=0}$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.



(a) State the critical points of f.

$\boxed{x = -4}$

(b) State the interval(s) on which f increases.

$\boxed{(-4,\infty)}$ because $f'(x) > 0$ there

(c) State the interval(s) on which f decreases.

$\boxed{(-\infty,-4)}$ because $f'(x) < 0$ there.

(d) Does f have a local maximum? Where?.

$\boxed{\text{No local max}}$

(e) Does f have a local minimum? Where?.

$\boxed{\text{Local min at } x = -4}$

Name: Richard

QUIZ 15 ◊

MATH 200

November 1, 2021

1. (10 points) This problem concerns the function $f(x) = e^{x^3-3x}$.

(a) Find the intervals on which f increases and on which it decreases.

$f'(x) = e^{x^3-3x}(3x^2-3) = e^{x^3-3x}3(x^2-1) = e^{x^3-3x}3(x-1)(x+1)$

$x=1 \quad x=-1$

The critical points are $x=1$ and $x=-1$.

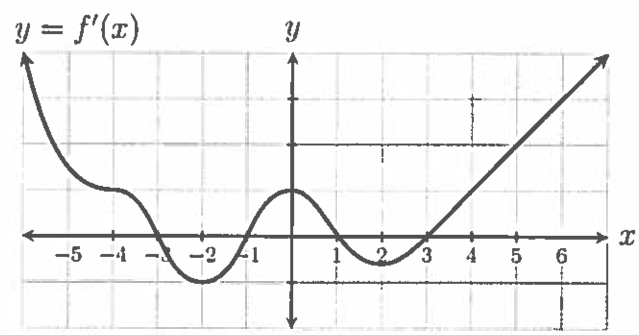
	-1		1	
$+ + + + +$		$+ + + + + + +$		$+ + + +$ e^{x^3-3x}
$- - - - -$		$- - - - -$		$+ + + +$ $3(x-1)$
$- - - - -$		$+ + + + + + +$		$+ + + +$ $(x+1)$
$+ + + + +$		$- - - - - - -$		$+ + + +$ $f'(x) = e^{x^3-3x}3(x-1)(x+1)$

$\boxed{f \text{ increases on } (-\infty,-1) \,\&\, (1,\infty)}$ $\boxed{f \text{ decreases on } (-1,1)}$

(b) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

By the first derivative test, There is a $\boxed{\text{local maximum at } x=-1}$ and a $\boxed{\text{local minimum at } x=1}$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.



(a) State the critical points of f.

$-3, -1, 1, 3$

(b) State the interval(s) on which f increases.

$(-\infty,-3) \,\&\, (-1,1) \,\&\, (3,\infty)$

(c) State the interval(s) on which f decreases.

$(-3,-1) \,\&\, (1,3)$

(d) Does f have a local maximum? Where?.

Yes at $x=-3$ and at $x=1$ (By 1st derivative test)

(e) Does f have a local minimum? Where?.

Yes at $x=-1$ and at $x=3$ (By 1st derivative test)

1. (10 points) This problem concerns the function $f(x) = 5x^4 + 20x^3 + 10$.

(a) Find the intervals on which f increases and on which it decreases.

$f'(x) = 20x^3 + 60x^2 = 20x^2(x+3) = 0$

$x = 0$ $x = -3$

The critical points are $x=0$ and $x=-3$

	-3		0		
$++++$	|	$+ ++++$	|	$++++$	x^2
$----$	|	$+ ++++$	|	$++++$	$(x+3)$
$----$	|	$+ ++++$	|	$++++$	$f'(x) = 20x^2(x+3)$

$\boxed{f(x) \text{ increases on } (-3, \infty)}$ and $\boxed{f(x) \text{ decreases on } (-\infty, -3)}$

(b) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

By the first derivative test

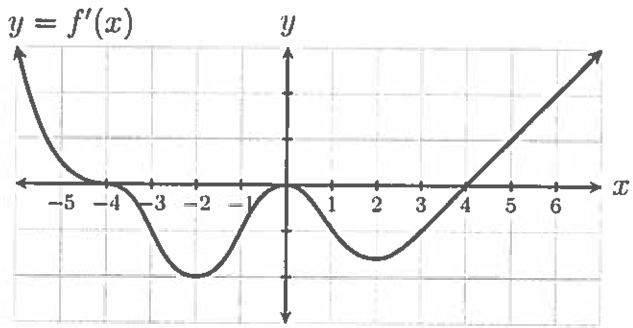
$\boxed{f(x) \text{ has a local minimum at } x = -3}$

There is $\boxed{\text{no local maximum}}$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.

(a) State the critical points of f.

$-4, 0, 4$



(b) State the interval(s) on which f increases.

$(-\infty, -4)$ & $(4, \infty)$

(c) State the interval(s) on which f decreases.

$(-4, 0)$ & $(0, 4)$

(d) Does f have a local maximum? Where?.

$\boxed{\text{Local max at } x = -4}$ (Because $f'(x)$ changes from $+$ to $-$ at -4)

(e) Does f have a local minimum? Where?.

$\boxed{\text{Local min at } x = 4}$ (Because $f'(x)$ changes from $-$ to $+$ at $x=4$)