Name: Richard

QUIZ 15 ♡

MATH 200
October 31, 2022

1. (12 points) This problem concerns the function $f(x) = 60x - 9x^2 - 2x^3$.

(a) Find the critical points.

$$f'(x) = 60 - 18x - 6x^2$$
$$= -6(x^2 + 3x - 10)$$
$$= -6(x-2)(x+5)$$

$\boxed{x=2}$ $\boxed{x=-5}$

Critical points are $x=2$ and $x=-5$

(b) Find the intervals on which f increases and on which it decreases.

	-5		2		
$- - -$		$- - - - - - -$		$- - - - -$	-6
$- - -$		$- - - - -$		$+ + + +$	$(x-2)$
$- - -$		$+ + + +$		$+ + + +$	$(x+5)$
$- - -$		$+ + + + +$		$- - - -$	$f'(x) = -6(x-2)(x-5)$

f decreases on $(-\infty, -5) \cup (2, \infty)$

f increases on $(-5, 2)$

(c) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

local minimum at $x = -5$
local maximum at $x = 2$

2. (8 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below.

local max

(a) State the critical points of f.

0 and 5

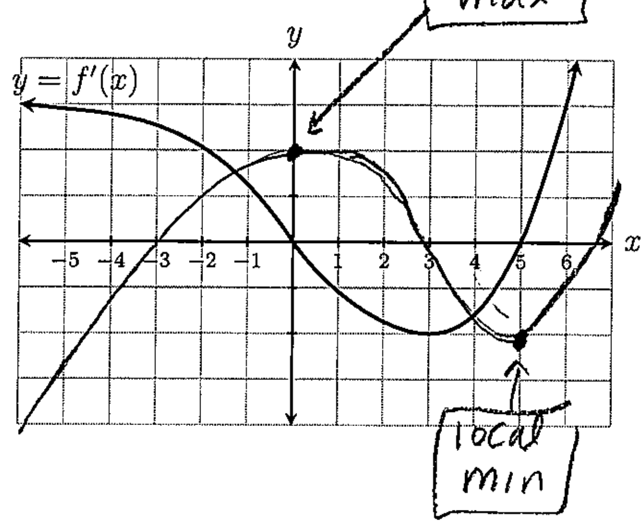
(b) State the interval(s) on which f increases.

$(-\infty, 0) \cup (5, \infty)$ because $f'(x) > 0$ there

(c) State the interval(s) on which f decreases.

$(0, 5)$ because $f'(x) < 0$ there

(d) Using the same coordinate axes, sketch a possible graph of $y = f(x)$.
Be sure to clearly indicate any local extrema.



Name: Richard	QUIZ 15 ♣	MATH 200 October 31, 2022

1. (12 points) This problem concerns the function $f(x) = x^2e^x - 3e^x$.

   (a) Find the critical points.

$$f'(x) = 2xe^x + x^2e^x - 3e^x$$
$$= e^x(x^2 + 2x - 3)$$
$$= e^x(x-1)(x+3) = 0$$

$\boxed{x=1}$ $\boxed{x=-3}$

Critical points are $x = 1$ and $x = -3$

   (b) Find the intervals on which f increases and on which it decreases.

	-3		1		
+ + + + +		+ + + +		+ + + + +	e^x
− − −		− − − −		+ + + + +	$(x-1)$
− − −		+ + + +		+ + + + +	$(x+3)$
+ + + +		− − − −		+ + + + +	$f'(x) = e^x(x-1)(x+1)$

f increases on $(-\infty, -3)$ & $(1, \infty)$

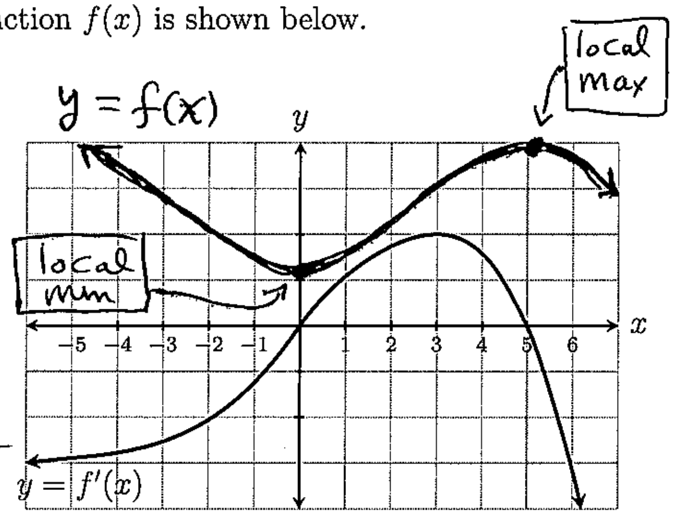
f decreases on $(-3, 1)$

   (c) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

By First Derivative Test:

f has a local maximum at $x = -3$
f has a local minimum at $x = 1$

2. (8 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below.



   (a) State the critical points of f.

$\boxed{0, 5}$

   (b) State the interval(s) on which f increases.

$\boxed{(0, 5)}$ because $f'(x) > 0$ there.

   (c) State the interval(s) on which f decreases.

$\boxed{(-\infty, 0) \cup (5, \infty)}$ as $f'(x) < 0$ there

   (d) Using the same coordinate axes, sketch a possible graph of $y = f(x)$.
   Be sure to clearly indicate any local extrema.

Name: Richard

QUIZ 15 ♢

MATH 200
October 31, 2022

1. (12 points) This problem concerns the function $f(x) = \ln(x^2 - 6x + 10)$.

   (a) Find the critical points.

$$f'(x) = \frac{2x-6}{x^2-6x+10} = \frac{2(x-3)}{x^2-6x+9+1} = \frac{2(x-3)}{(x-3)^2+1}$$

equals 0 only for $x=3$

always positive

$f'(x)$ is defined for all values of x and it equals 0 only when $x=3$. Therefore $\boxed{x = 3 \text{ is the only critical pt.}}$

   (b) Find the intervals on which f increases and on which it decreases.

– – – – – – – – – – 3 + + + + + + $2(x-3)$

+ + + + + + + + + + + + + + + + $(x-3)^2+1$

– – – – – – – – | + + + + + + $f'(x) = \frac{2(x-3)}{(x-3)^2+1}$

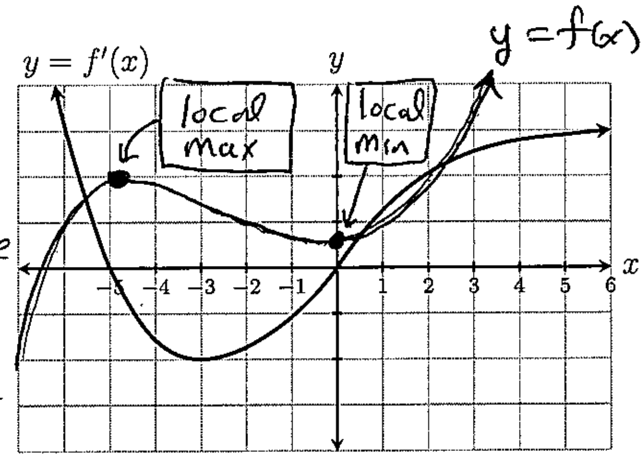
$\boxed{f(x) \text{ decreases on } (-\infty, 3)}$ $\boxed{f(x) \text{ increases on } (3, \infty)}$

   (c) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

By the first derivative test $f(x)$ has a $\boxed{\text{local minimum at } x=3}$

There is $\boxed{\text{no local maximum}}$

2. (8 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below.



   (a) State the critical points of f.

–5 and 0

   (b) State the interval(s) on which f increases.

$\boxed{(-\infty, -5) \cup (0, \infty)}$ as $f'(x) > 0$ there

   (c) State the interval(s) on which f decreases.

$\boxed{(-5, 0)}$ because $f'(x) < 0$ there

   (d) Using the same coordinate axes, sketch a possible graph of $y = f(x)$.
   Be sure to clearly indicate any local extrema.

Name: Richard

QUIZ 15 ♠

MATH 200
October 31, 2022

1. (12 points) This problem concerns the function $f(x) = 3x^4 + 4x^3 - 2$.

(a) Find the critical points.

$$f'(x) = 12x^3 + 12x^2 = 12x^2(x+1)$$

$$x = 0 \qquad x = 1$$

$\boxed{\text{Critical points } x=0 \text{ and } x=-1}$

(b) Find the intervals on which f increases and on which it decreases.

	-1	0	
+ + + +	+ + + +	+ + + + +	$12x^2$
– – – –	+ + + +	+ + + + +	$(x+1)$
– – – –	+ + + +	+ + + + +	$f'(x) = 12x^2(x+1)$

$\boxed{f \text{ decreases on } (-\infty,-1)}$ $\boxed{f \text{ increases on } (-1,\infty)}$

(c) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

f has a $\boxed{\text{local minimum at } x = -1}$

There is $\boxed{\text{no local maximum}}$

2. (8 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below.

(a) State the critical points of f.

0, 5

(b) State the interval(s) on which f increases.

$(-\infty,0) \cup (5,\infty)$ since $f'(x) >$ there

(c) State the interval(s) on which f decreases.

$(0,5)$ because $f'(x) < 0$ there

(d) Using the same coordinate axes, sketch a possible graph of $y = f(x)$.
Be sure to clearly indicate any local extrema.