Name: Richard

QUIZ 15

MATH 200
October 12, 2023

1. (10 points) This problem concerns the function $f(x) = \sqrt[3]{8 - x^3}$. $= (8-x^3)^{1/3}$

   (a) Find the critical points of f.

$$f'(x) = \frac{1}{3}(8-x^3)^{-2/3}(0-3x^2) = \frac{-3x^2}{3(8-x^3)^{2/3}}$$

$$= -\frac{x^2}{\sqrt[3]{8-x^3}^{\,2}}$$

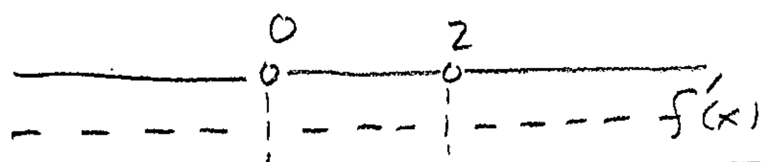
$$= -\left(\frac{x}{\sqrt[3]{8-x^3}}\right)^2$$

Notice that $f'(x) = -\left(\frac{x}{\sqrt[3]{8-x^2}}\right)^2$ equals 0 only when $x=0$ (making the numerator 0) and $f'(x)$ is undefined for $x=2$ (making the denominator 0). Therefore

$\boxed{\text{the critical points are } x=0 \text{ and } x=2}$

   (b) Find the intervals on which f increases and on which it decreases.

Notice that $f'(x) = -\left(\frac{x}{\sqrt[3]{8-x^3}}\right)^2$ is negative for all values of x that are not critical points



Thus $\boxed{f(x) \text{ decreases on } (-\infty,0)\cup(0,2)\cup(2,\infty)}$

   (c) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

Because f never switches increase/decrease

$\boxed{\text{there are no extrema}}$

Name: Richard

QUIZ 15

MATH 200
October 12, 2023

1. (10 points) This problem concerns the function $f(x) = \tan^{-1}(x^2 + x - 2)$.

   (a) Find the critical points of f.

$$f'(x) = \frac{1}{1+(x^2+x-2)^2}(2x+1-0) = \frac{2x+1}{1+(x^2+x-2)^2}$$

Notice that the denominator is always positive for any values of x. (It is 1 plus a number squared). Thus $f'(x)$ is defined for all x and $f'(x)=0$ only for $x = -1/2$ (which makes the numerator 0)

Thus $\boxed{x = -1/2 \text{ is the only critical point}}$

   (b) Find the intervals on which f increases and on which it decreases.

$-1/2$		0	
− − − −	+ + + + + + + + + +		$2x+1$
+ + + +	+ + + + + + + + +		$1+(x^2+x-2)^2$
− − −	+ + + + + + + +		$f'(x) = \dfrac{2x+1}{1+(x^2+x-2)^2}$

f decreases on $(-\infty, -1/2)$
f increases on $(-1/2, \infty)$

   (c) Use your answer from part (a) to identify the locations (x values) of any local extrema of f.

By the first derivative test f has a local minimum at $x = -1/2$.

There is no local maximum