Name: Richard

QUIZ 16 ♡

MATH 200
November 3, 2021

1. (10 points) Use the second derivative test to find the local extrema of $f(x) = x^3 + 3x^2 + 10$.

$$f'(x) = 3x^2 + 6x$$
$$= 3x(x+2) = 0$$

$x = 0$ $\quad$ $x = -2$ ← critical points

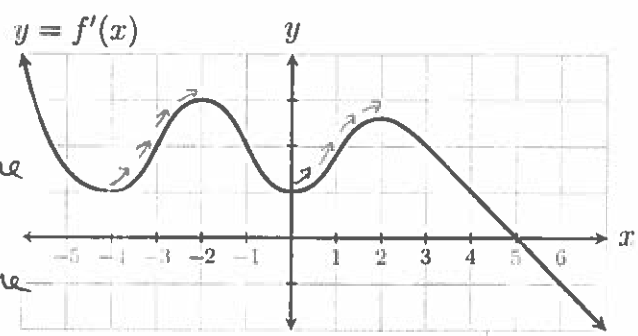
$$f''(x) = 6x + 6$$

Test $x=0$: $f''(0) = 6 \cdot 0 + 6 = 6 > 0$ $\boxed{\text{so local min at } x=0}$

Test $x=-2$: $f''(-2) = 6(-2)+6 = -6 < 0$ so $\boxed{\text{local max at } x=-2}$

Ans: Local min of $f(0) = 10$ at $x=0$
Local max of $f(-2) = 14$ at $x=-2$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below. Answer the following questions about the function $f(x)$.



(a) State the critical points of f.

$\boxed{x = 5}$

(b) State the interval(s) on which f increases.

$\boxed{(-\infty, 5)}$ because $f'(x) > 0$ there

(c) State the interval(s) on which f decreases.

$\boxed{(5, \infty)}$ because $f'(x) < 0$ there

(d) State the intervals on which f is concave up.

$\boxed{(-4,-2) \text{ and } (0,2)}$ because $f'(x)$ increases there, so $f''(x) > 0$

(e) State the intervals on which f is concave down.

$\boxed{(-\infty,-4) \text{ and } (-2,0) \text{ and } (2,\infty)}$ (because $f'(x)$ decreases there, so $f''(x) < 0$)

Name: Richard

QUIZ 16 ♣

MATH 200
November 3, 2021

1. (10 points) Use the second derivative test to find the local extrema of $f(x) = xe^x + e^x$.

$f'(x) = 1 \cdot e^x + xe^x + e^x$
$= xe^x + 2e^x$
$= e^x(x+2) = 0$

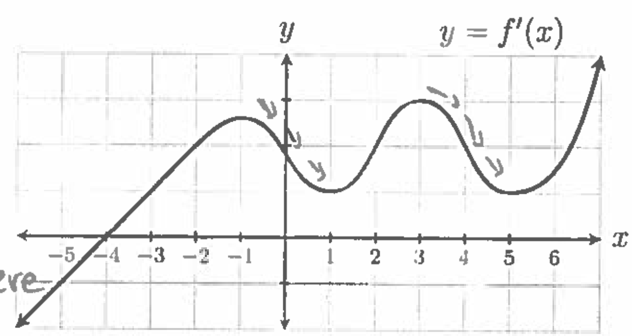
$x = -2$ ← $\boxed{\text{critical point}}$

$f''(x) = 1 \cdot e^x + xe^x + 2e^x = xe^x + 3e^x$

Test $x=-2$: $f''(-2) = -2e^{-2} + 3e^{-2} = e^{-2} = \frac{1}{e^2} > 0$

Thus there is a local minimum of
$f(-2) = -2e^{-2} + e^{-2} = -e^{-2} = \frac{-1}{e^2}$ at $x = -2$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below.
Answer the following questions about the function $f(x)$.



(a) State the critical points of f.

$\boxed{-4}$ because $f'(-4) = 0$

(b) State the interval(s) on which f increases.

$\boxed{(-4, \infty)}$ because $f'(x) > 0$ there

(c) State the interval(s) on which f decreases.

$\boxed{(-\infty, -4)}$ because $f'(x) < 0$ there

(d) State the intervals on which f is concave up.

$(-\infty, -1)$ $(1, 3)$ $(5, \infty)$ ← because $f'(x)$ increases there, so $f''(x) > 0$

(e) State the intervals on which f is concave down.

$(-1, 1)$ $(3, 5)$ ← because $f'(x)$ decreases there

Name: Richard

QUIZ 16 ◇

MATH 200
November 3, 2021

1. (10 points) Use the second derivative test to find the local extrema of $f(x) = e^{x^2-2x}$.

$$f'(x) = e^{x^2-2x}(2x-2) = 2e^{x^2-2x}(x-1) = 0$$

$x=1$ is critical point

$$f''(x) = 2e^{x^2-2x}(2x-2)(x-1) + 2e^{x^2-2x}(1)$$

$$f''(x) = 4e^{x^2-2x}(x-1)^2 + 2e^{x^2-2x}$$

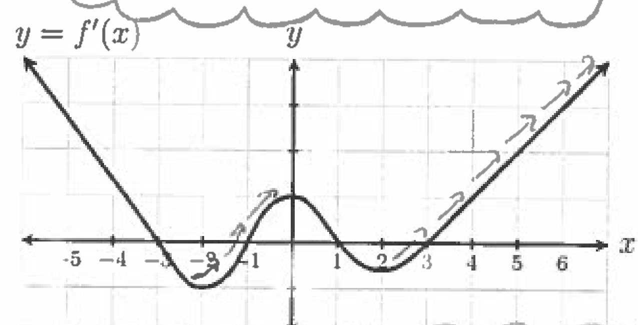
Test $x=1$: $f''(1) = 4e^{1^2-2\cdot 1}(1-1)^2 + 2e^{1^2-2\cdot 1}$

$$= 4e^{-1}\cdot 0^2 + 2e^{-1} = \frac{2}{e} > 0$$

$\boxed{\text{Thus } f(x) \text{ has a local minimum of } f(x) = e^{1^2-2\cdot 1} = e^{-1} = \frac{1}{e} \text{ at } x = 1}$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below. Answer the following questions about the function $f(x)$.

$f'(c)=0$ for these c



(a) State the critical points of f.

$-3,\ -1,\ 1,\ 3$

(b) State the interval(s) on which f increases.

$(-\infty, -3)\ (-1, 1)\ (3, \infty)$

(c) State the interval(s) on which f decreases.

$(-3, -1)\ (1, 3)$

(d) State the intervals on which f is concave up.

$(-2, 0)\ (2, \infty)$ ← because $f'(x)$ increases here, so $f''(x) > 0$ on these intervals

(e) State the intervals on which f is concave down.

$(-\infty, -2)\ (0, 2)$ ← because $f'(x)$ decreases here, so $f''(x) < 0$ on these intervals

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QUIZ 16 ♠

MATH 200
November 3, 2021

1. (10 points) Use the second derivative test to find the local extrema of $f(x) = xe^{-x}$.

$$f'(x) = 1\cdot e^{-x} + xe^{-x}(-1) = e^{-x} - xe^{-x} = e^{-x}(1-x) = 0$$

$\boxed{x = 1 \text{ is the only critical point}}$

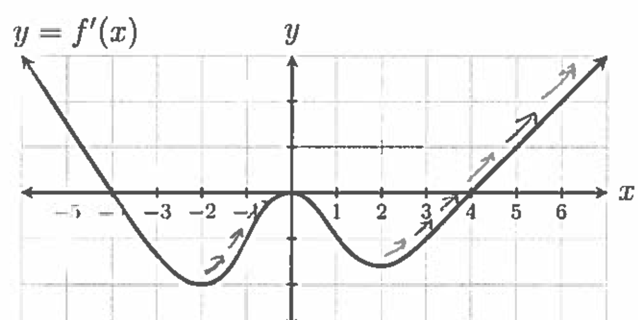
$$f''(x) = D_x\left[e^{-x} - xe^{-x}\right] = e^{-x}(-1) - 1\cdot e^{-x} - xe^{-x}(-1)$$
$$= xe^{-x} - 2e^{-x}$$

Test $x=1$: $f''(1) = 1\cdot e^{-1} - 2e^{-1} = -e^{-1} = -\frac{1}{e} < 0$

Therefore there is a $\boxed{\text{local maximum at } x = 1.}$
The local maximum is $f(1) = 1\cdot e^{-1} = 1/e$.

No local minima.

2. (10 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below. Answer the following questions about the function $f(x)$.



(a) State the critical points of f.

$-4, \ 0, \ 4$

(b) State the interval(s) on which f increases.

$(-\infty, -4)$ and $(4, \infty)$

(c) State the interval(s) on which f decreases.

$(-4, 0)$ and $(0, 4)$

(d) State the intervals on which f is concave up.

$(-2, 0)$ and $(2, \infty)$ ← because $f'(x)$ increases there, so $f''(x) > 0$

(e) State the intervals on which f is concave down.

$(-\infty, -2)$ and $(0, 2)$ ← because $f'(x)$ decreases there, so $f''(x) < 0$