Name: Richard

QUIZ 16 ♡

MATH 200
November 2, 2021

This quiz concerns the function $f(x) = 3x + \dfrac{75}{x} + 10$.

1. (10 points) Use the second derivative test to find the local extrema of $f(x)$.

$$f'(x) = 3 - \frac{75}{x^2} + 0 = 0$$
$$3 = \frac{75}{x^2}$$
$$3x^2 = 75$$
$$x^2 = 25$$
$$x = \pm\sqrt{25} = \pm 5$$

critical points are $x=5$ and $x=-5$.

$$f''(x) = \frac{150}{x^3}$$

Check $x=5$: $f''(5) = \frac{150}{5^3} > 0$ ← local min

Check $x=-5$: $f''(-5) = \frac{150}{(-5)^3} < 0$ ← local max

f has a local minimum of $f(5) = 40$ at $x = 5$
f has a local maximum of $f(-5) = -20$ at $x = -5$

2. (5 points) Find the interval(s) on which $f(x)$ is concave up.

0

$- - - - \mid + + +$ $\quad f''(x) = \frac{150}{x^3}$

f is concave up on $(0, \infty)$

3. (5 points) Find the interval(s) on which $f(x)$ is concave down.

f is concave down on $(-\infty, 0)$

Name: Richard

QUIZ 16 ♣

MATH 200
November 2, 2021

This quiz concerns the function $f(x) = 2x + \dfrac{8}{x^2}$.

1. (10 points) Use the second derivative test to find the local extrema of $f(x)$.

$$f'(x) = 2 - \frac{16}{x^3} = 0$$

$$2 = \frac{16}{x^3}$$

$$2x^3 = 16$$

$$x^3 = 8$$

$$x = \sqrt[3]{8} = 2$$

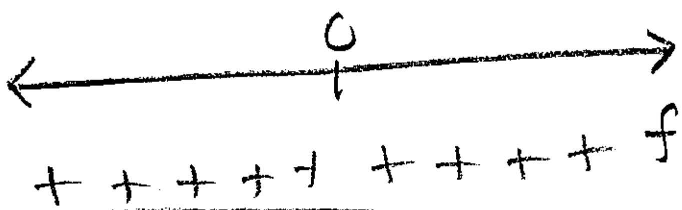
The only critical point is $x = 2$

$$f''(x) = \frac{48}{x^4}$$

$$f''(2) = \frac{48}{2^4} = \frac{48}{16} = 3 > 0$$

f has a local minimum of $f(2) = 6$ at $x = 2$
f has no local maximum

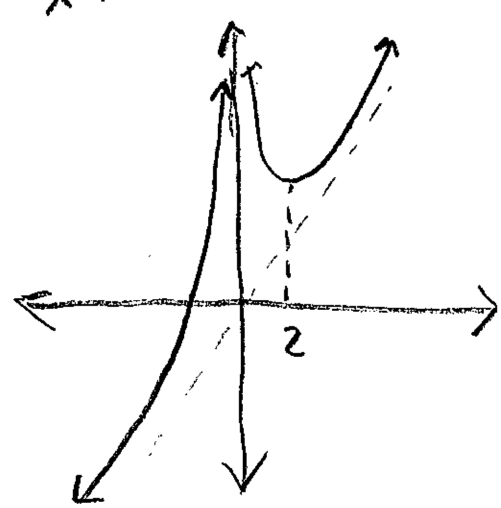
2. (5 points) Find the interval(s) on which $f(x)$ is concave up.



$$f''(x) = \frac{48}{x^4}$$

Concave up: $(-\infty, 0) \cup (0, \infty)$

3. (5 points) Find the interval(s) on which $f(x)$ is concave down.



never concave down

Name: Richard

Quiz 16 ◊

MATH 200
November 2, 2021

This quiz concerns the function $f(x) = 100 + 300x - x^3$.

1. (10 points) Use the second derivative test to find the local extrema of $f(x)$.

$$f'(x) = 300 - 3x^2 = 0$$
$$-3x^2 = -300$$
$$x^2 = 100$$
$$x = \pm\sqrt{100} = \pm 10$$

The two critical points are $x=10$ and $x=-10$

$$f''(x) = -6x$$

<u>Check $x=10$</u> $f''(10) = -6 \cdot 10 = -60 < 0$ (local max)

<u>Check $x=-10$</u> $f''(-10) = -6(-10) = 60 > 0$ (local min)

$f(x)$ has a <u>local max</u> of $f(10) = 2100$ at $x = 10$
$f(x)$ has a <u>local min</u> of $f(-10) = -1900$ at $x = -10$

2. (5 points) Find the interval(s) on which $f(x)$ is concave up.

0

+ + + + + + | − − − − − $f''(x) = -6x$

$f(x)$ is concave up on $(-\infty, 0)$

3. (5 points) Find the interval(s) on which $f(x)$ is concave down.

$f(x)$ is concave down on $(0, \infty)$

Name: Richard

QUIZ 16 ♠

MATH 200
November 2, 2021

This quiz concerns the function $f(x) = x^3 - 75x + 10$.

1. (10 points) Use the second derivative test to find the local extrema of $f(x)$.

$$f'(x) = 3x^2 - 75 = 0$$
$$3x^2 = 75$$
$$x^2 = 25$$
$$x = \pm\sqrt{25} = \pm 5$$

The critical points are $x = 5$ and $x = -5$

$$f''(x) = 6x$$

Check $x=5$: $f''(5) = 6\cdot 5 = 30 > 0$ (local min)

Check $x=-5$: $f''(-5) = 6(-5) = -30 < 0$ (local max)

$f(x)$ has a local min of $f(5) = -240$ at $x = 5$
$f(x)$ has a local max of $f(-5) = 260$ at $x = -5$

2. (5 points) Find the interval(s) on which $f(x)$ is concave up.

0

– – – – – | + + + + + $f''(x) = 6x$

$f(x)$ is concave up on $(0, \infty)$

3. (5 points) Find the interval(s) on which $f(x)$ is concave down.

$f(x)$ is concave down on $(-\infty, 0)$