Name: Richard

Quiz 16

MATH 200
October 22, 2024

1. (10 points) Use the second derivative test to find the local extrema of $f(x) = \frac{x^3}{3} + x^2 + 1$.

$f'(x) = x^2 + 2x = x(x+2) = 0$

Critical points: $x = 0 \quad x = -2$

$f''(x) = 2x + 2$

Test $x = 0$: $f''(0) = 2 \cdot 0 + 2 = 2 > 0$, so local min at $x = 0$

Test $x = -2$: $f''(-2) = 2(-2) + 2 = -2 < 0$, so local max at $x = -2$

Answer: $f(x)$ has a local min of $f(0) = 1$ at $x = 0$.
$f(x)$ has a local max of $f(-2) = \frac{7}{3}$ at $x = -2$

2. (10 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below. Answer the following questions about the function $f(x)$.

(a) State the critical points of f.

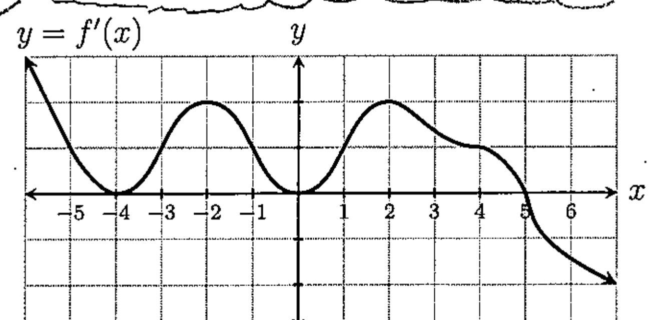
$x = -4, \; x = 0, \; x = 5$

Because that's where $f'(x) = 0$

(b) State the interval(s) on which f increases.

$(-\infty, -4) \cup (-4, 0) \cup (0, 5)$

Because that's where $f'(x) > 0$



(c) State the interval(s) on which f decreases.

$(5, \infty)$

(d) State the intervals on which f is concave up.

$(-4, -2) \cup (0, 2)$ ← That's where $f'(x)$ increases.

(e) State the intervals on which f is concave down.

$(-\infty, -4) \cup (-2, 0) \cup (2, \infty)$ ← That's where $f'(x)$ decreases

Name: Richard

Quiz 16

MATH 200
October 22, 2024

1. (10 points) Use the second derivative test to find the local extrema of $f(x) = xe^x$.

$f'(x) = 1 \cdot e^x + xe^x = e^x(1+x) = 0$

Critical point: $x = -1$

$f''(x) = e^x + 1e^x + xe^x = 2e^x + xe^x$

Test. $x = -1$ $\quad f''(-1) = 2e^{-1} + (-1)e^{-1} = \frac{2}{e} - \frac{1}{e} = \frac{1}{e} > 0.$

Therefore there is a local minimum of $f(-1) = \frac{-1}{e}$ at $x = -1$. No local maximum

2. (10 points) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is shown below. Answer the following questions about the function $f(x)$.

(a) State the critical points of f.

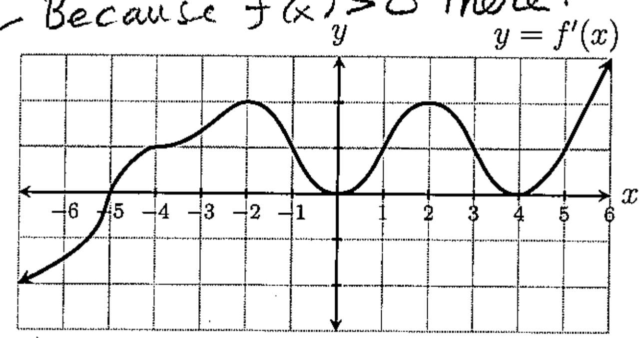
$-5, 0, 4$

Because that's where $f'(x) = 0$

(b) State the interval(s) on which f increases.

$(-5,0) \cup (0,4) \cup (4,\infty)$

Because $f'(x) > 0$ there.



(c) State the interval(s) on which f decreases.

$(-\infty, -5)$

(d) State the intervals on which f is concave up.

$(-\infty,-2) \cup (0,2) \cup (4,\infty)$

Because that's where $f'(x)$ increases

(e) State the intervals on which f is concave down.

$(-2,0) \cup (2,4)$

Because that's where $f'(x)$ decreases