Name: Richard

Quiz 16

MATH 200
October 21, 2025

1. This problem concerns the function $f(x) = x^2 e^{-x}$.

(a) $f'(x) = 2xe^{-x} + x^2 e^{-x}(-1)$

$$\boxed{f'(x) = e^{-x}(2x - x^2)}$$

(b) Find the critical points of f

$$f'(x) = 0$$
$$e^{-x}(2x - x^2) = 0$$
$$e^{-x}x(2-x) = 0$$

Critical points: $\boxed{x=0 \qquad x=2}$

(c) $f''(x) = e^{-x}(-1)(2x - x^2) + e^{-x}(2 - 2x)$

$$= e^{-x}(-2x + x^2 + 2 - 2x)$$

$$\boxed{f''(x) = e^{-x}(x^2 - 4x + 2)}$$

(d) Use the second derivative test to identify the local extrema of f.

Test $x=0$: $f''(0) = e^{-0}(0^2 - 4\cdot 0 + 2) = 1\cdot 2 = 2 > 0$

$\therefore$ $\boxed{f \text{ has a local minimum at } x = 0}$

Test $x=2$: $f''(2) = e^{2}(2^2 - 4\cdot 2 + 2) = -2e^2 < 0$

$\therefore$ $\boxed{f \text{ has a local maximum at } x = 2}$

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QUIZ 16

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1. This problem concerns the function $f(x) = 4 - x^2e^x$.

(a) $f'(x) = 0 - 2xe^x - x^2e^x$

$$\boxed{f'(x) = -e^x(2x + x^2)}$$

(b) Find the critical points of f

$$f'(x) = 0$$

$$-e^x(2x + x^2) = 0$$

$$-e^x x(2 + x) = 0$$

Critical points: $\boxed{x = 0 \qquad x = -2}$

(c) $f''(x) = -e^x(2x + x^2) - e^x(2 + 2x)$

$$f''(x) = -e^x(2x + x^2 + 2 + 2x) = -e^x(x^2 + 4x + 2)$$

$$\boxed{f''(x) = -e^x(x^2 + 4x + 2)}$$

(d) Use the second derivative test to identify the local extrema of f.

Test $x=0$: $f''(0) = -e^0(0^2 + 4 \cdot 0 + 2) = -1 \cdot 2 = -2 < 0$

$\therefore$ $\boxed{\text{Local maximum at } x = 0}$

Test $x=-2$: $f''(-2) = -e^{-2}((-2)^2 + 4(-2) + 2) = -\frac{-2}{e^2} = \frac{2}{e^2} > 0$

$\therefore$ $\boxed{\text{Local minimum at } x = -2}$