Name: Richard

QUIZ 17 ♠

MATH 200
November 8, 2021

1. (10 points) Find the global extrema of the function $f(x) = x\sqrt{2-x}$ on the closed interval $[-2, 2]$.

$f(x) = x(2-x)^{1/2}$

$f'(x) = 1 \cdot \sqrt{2-x} + x\frac{1}{2}(2-x)^{-1/2}(-1)$

$f'(x) = \sqrt{2-x} - \dfrac{x}{2\sqrt{2-x}}$ ← $f'(2)$ undefined, but $x=2$ is an endpoint

Solve $f'(x)=0$

$\sqrt{2-x} - \dfrac{x}{2\sqrt{2-x}} = 0$

$\sqrt{2-x} = \dfrac{x}{2\sqrt{2-x}}$

$2\sqrt{2-x}^{\,2} = x$

$2(2-x) = x$

$4-2x = x$

$4 = 3x$

$x = \frac{4}{3}$

critical point

$f(-2) = -2\sqrt{2-(-2)} = -2\sqrt{4} = -4$ ← MIN

$f(2) = 2\sqrt{2-2} = 2\sqrt{0} = 0$

$f(4/3) = \frac{4}{3}\sqrt{2-\frac{4}{3}} = \frac{4}{3}\sqrt{\frac{2}{3}} \approx 0.8$ ← MAX

Answer:
f has a global max of $\frac{4}{3}\sqrt{\frac{2}{3}}$ at $x=\frac{4}{3}$
f has a global min of -4 at $x=-2$

2. (10 points) Find the global extrema of the function $f(x) = x^2 + \dfrac{16}{x}$ on the open interval $(0, \infty)$.

$f'(x) = 2x - \dfrac{16}{x^2}$ ← Note: $f'(0)$ not defined, but 0 is not a critical point because it's not in the domain of $f(x)$

To find critical points, solve $f'(x)=0$

$2x - \dfrac{16}{x^2} = 0$

$2x = \dfrac{16}{x^2}$

$2x^3 = 16$

$x^3 = 8$

$x = \sqrt[3]{8} = 2$

Thus $x=2$ is a critical point, the only one in $(0,\infty)$

$f''(x) = 2 + \dfrac{32}{x^3}$

$f''(2) = 2 + \dfrac{32}{2^3} > 0$ so f has a local minimum at $x=2$.
Because 2 is the only critical point in $(0,\infty)$, this is a global min.

Answer: f has a global minimum of $f(2)=12$ at $x=2$. There is no global maximum.

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QUIZ 17 ♡

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1. (10 points) Find the global extrema of the function $f(x) = x^3 - 3x$ on the closed interval $[0, 2]$.

$$f'(x) = 3x^2 - 3x = 3(x^2-1) = 3(x-1)(x+1) = 0$$

Critical points: $x = 1$ $\quad x = -1$

The only critical point in the interval is $x = 1$

$f(1) = 1^3 - 3\cdot 1 = -2 \leftarrow$ MIN

$f(0) = 0^3 - 3\cdot 0 = 0$

$f(2) = (2)^3 - 3(2) = 2 \leftarrow$ MAX

Answer: f has a global minimum of -2 at $x=1$
f has a global maximum of 2 at $x=2$

2. (10 points) Find the global extrema of the function $f(x) = xe^{3x}$ on the open interval $(-5, \infty)$.

$$f'(x) = 1\cdot e^{3x} + xe^{3(x)}(3) = e^{3x}(1+3x) = 0$$

$x = -\frac{1}{3}$

There is only one critical point in $(-5, \infty)$ namely $x = -\frac{1}{3}$

-5 - - - - $-\frac{1}{3}$ + + + + $\quad f'(x) = e^{3x}(1+3x)$

By the first derivative test f has a local minimum at $x = -\frac{1}{3}$. Since $-\frac{1}{3}$ was the only critical point, this is a global minimum.

Ans f has a global minimum at $x = -\frac{1}{3}$. No global max.

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QUIZ 17 ♣

MATH 200
November 8, 2021

1. (10 points) Find the global extrema of the function $f(x) = x + \frac{1}{x}$ on the closed interval $\left[\frac{1}{2}, 3\right]$.

$f'(x) = 1 - \frac{1}{x^2}$ Notice $f'(0)$ is undefined, but 0 is not a critical point because 0 is not in the domain of f

Thus to find all critical points we solve $f'(x) = 0$

$1 - \frac{1}{x^2} = 0$

$1 = \frac{1}{x^2}$

$x^2 = 1$

$x = \pm 1 \leftarrow$ So the only critical point in $\left[\frac{1}{2}, 3\right]$ is $x = 1$

$$\left.\begin{array}{l} f(\frac{1}{2}) = \frac{1}{2} + \frac{1}{1/2} = \frac{5}{2} = 2.5 \\ f(1) = 1 + \frac{1}{1} = 2 \\ f(3) = 3 + \frac{1}{3} = \frac{10}{3} = 3.\overline{3} \end{array}\right\}$$

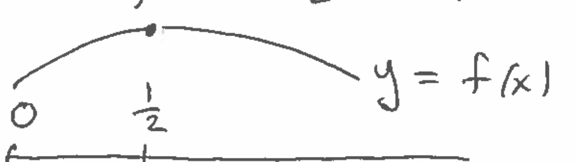
Answer:
f has a global max of $f(3) = \frac{10}{3}$ at $x=3$
f has a global min of $f(1) = 2$ at $x = 1$

2. (10 points) Find the global extrema of the function $f(x) = xe^{-2x}$ on the open interval $(0, \infty)$.

$f'(x) = 1e^{-2x} + xe^{-2x}(-2) = e^{-2x}(1-2x) = 0$

$x = \frac{1}{2}$ is the critical point

So the interval $(0, \infty)$ contains only one critical point, namely $x = \frac{1}{2}$. By the first derivative test f has a local maximum at $x = \frac{1}{2}$.



$+ + + \mid - - - - - - f'(x) = e^{-2x}(1-2x)$

Because there is only one critical point in $(0, \infty)$ this local maximum is a global maximum.

Answer $f(x)$ has a global maximum at $x = \frac{1}{2}$
There is no global minimum

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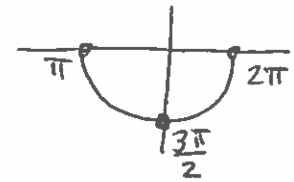
QUIZ 17 ◊

MATH 200
November 8, 2021

1. (10 points) Find the global extrema of the function $f(x) = \sin^2(x)$ on the closed interval $[\pi, 2\pi]$.

$$f'(x) = 2\sin(x)\cos(x) = 0$$



$x = \pi, 2\pi$ $\qquad$ $x = \frac{3\pi}{2}$

The critical points in $[\pi, 2\pi]$ are π, 2π and $\frac{3\pi}{2}$ (and two of these just happen to be endpoints as well)

$f(\pi) = \sin^2(\pi) = 0^2 = 0$
$f(2\pi) = \sin^2(2\pi) = 0^2 = 0$ } global min
$f\left(\frac{3\pi}{2}\right) = \sin^2\left(\frac{3\pi}{2}\right) = (-1)^2 = 1$ ← global max

Answer $f(x)$ has a global minimum of 0 at $x = \pi$ & 2π
$f(x)$ has a global maximum of 1 at $x = \frac{3\pi}{2}$

2. (10 points) Find the global extrema of the function $f(x) = 2x^2 + \frac{108}{x}$ on the open interval $(0, \infty)$.

$f'(x) = 4x - \frac{108}{x^2}$ $\quad$ To find the critical points, solve

$$4x - \frac{108}{x^2} = 0$$
$$4x = \frac{108}{x^2}$$
$$4x^3 = 108$$
$$x^3 = 27$$
$$x = \sqrt[3]{27} = 3$$

So the interval $(0, \infty)$ contains only one critical point, $x = 3$. To see if this gives a global max or min we'll use the second derivative test to find local extrema

$f''(x) = 4 + \frac{216}{x^3}$. Thus $f''(3) = 4 + \frac{216}{3} > 0$ and there is a local minimum at $x = 3$. Since this is the local extremum in $(0, \infty)$ it is a global minimum

Answer f has a global minimum of $f(3) = 2 \cdot 3^2 + \frac{108}{3} = 54$ at $x = 3$. There is no global maximum