Name: Richard

QUIZ 17 ♡

MATH 200
November 7, 2022

1. (10 points) Find the global extrema of the function $f(x) = 3x + \frac{75}{x} + 10$ on $(0, \infty)$.

$$f'(x) = 3 - \frac{75}{x^2} = 0$$
$$3 = \frac{75}{x^2}$$
$$3x^2 = 75$$
$$x^2 = 25$$
$$x = \pm\sqrt{25} = \pm 5$$

The critical points are $x=5$ and $x=-5$ but only $x=5$ is in the interval $(0,\infty)$

$$f''(x) = \frac{150}{x^3} \qquad f''(5) = \frac{150}{5^3} = \frac{150}{125} > 0 \leftarrow \text{minimum}$$

There is a local minimum at $x=5$, and because this is the only critical point in the interval,

$\boxed{f(x) \text{ has a global minimum at } x=5. \text{ No global max.}}$

2. (10 points) Find the global extrema of the function $f(x) = x^2 - 4x + 7$ on $[0, 3]$

$$f'(x) = 2x - 4 = 0$$
$$2x = 4$$
$$x = 2$$

critical point

$$f(0) = 0^2 - 4\cdot 0 + 7 = 7 \leftarrow \text{max}$$
$$f(2) = 2^2 - 4\cdot 2 + 7 = 3 \leftarrow \text{min}$$
$$f(3) = 3^2 - 4\cdot 3 + 7 = 4$$

$\boxed{\begin{array}{l} f(x) \text{ has a global maximum of } 7 \text{ at } x=0 \\ f(x) \text{ has a global minimum of } 3 \text{ at } x=2 \end{array}}$

Name: Richard

QUIZ 17 ♣

MATH 200

November 7, 2022

1. (10 points) Find the absolute extrema of the function $f(x) = 2x + \frac{8}{x^2}$ on $(0, \infty)$.

$$f'(x) = 2 - \frac{16}{x^3} = 0$$

$$2 = \frac{16}{x^3}$$

$$2x^3 = 16$$

$$x^3 = 8$$

$$x = \sqrt[3]{8} = 2$$

There is exactly one critical point $x = 2$ on The interval $(0, \infty)$

$$f''(x) = \frac{48}{x^4}$$

$f''(2) = \frac{48}{2^4} > 0$ so $f(x)$ has a local minimum at $x=2$.

Conclusion $\boxed{\text{$f(x)$ has a global minimum at $x = 2$; No global maximum}}$

2. (10 points) Find the absolute extrema of the function $f(x) = x^3 - 3x$ on $[0, 2]$.

$$f'(x) = 3x^2 - 3 = 0$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

f has two critical points $x = \pm 1$, but only $x=1$ is in The interval $[0,2]$

$f(0) = 0^3 - 3 \cdot 0 = 0$

$f(1) = 1^3 - 3 \cdot 1 = -2$ ← Global min

$f(2) = 2^3 - 3 \cdot 2 = 2$ ← Global max

$\boxed{\text{f has a global minimum of -2 at $x = 1$; f has a global maximum of 2 at $x = 2$}}$

Name: Richard

Quiz 17 ◇

MATH 200
November 7, 2022

1. (10 points) Find the absolute extrema of the function $f(x) = 100 + 300x - x^3$ on $(0, \infty)$.

$$f'(x) = 300 - 3x^2 = 0$$
$$300 = 3x^2$$
$$100 = x^2$$
$$x = \pm\sqrt{100} = \pm 10$$

f has two critical points $x = \pm 10$, but it has exactly one critical point $x = 10$ on $(0, \infty)$

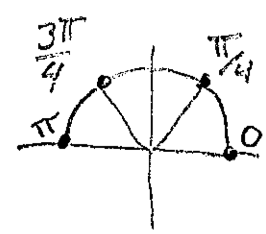
$$f''(x) = -6x^2$$
$$f''(10) = -6 \cdot 10^2 < 0 \leftarrow \text{local max.}$$

f has a local maximum at $x = 10$ but since this is the only critical point on the interval,
$\boxed{\text{f has a global maximum at } x=10. \text{ No global min.}}$

2. (10 points) Find the absolute extrema of the function $f(x) = \cos(x)\sin(x)$ on $[0, \pi]$.

$$f'(x) = -\sin(x)\sin(x) + \cos(x)\cos(x) = \cos^2(x) - \sin^2(x)$$
$$\cos^2(x) - \sin^2(x) = 0$$
$$\cos^2(x) = \sin^2(x)$$
$$\cos(x) = \pm \sin(x)$$

Critical points in $[0, \pi]$ are $\frac{\pi}{4}$ and $\frac{3\pi}{4}$



$$f(0) = \cos(0)\sin(0) = 1 \cdot 0 = 0$$
$$f(\pi/4) = \cos(\pi/4)\sin(\pi/4) = \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2} \leftarrow \text{global max}$$
$$f\left(\frac{3\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} = -\frac{2}{4} = \frac{-1}{2} \leftarrow \text{global min}$$
$$f(\pi) = \cos(\pi)\sin(\pi) = 1 \cdot 0 = 0$$

f has a global max of $\frac{1}{2}$ at $x = \pi/4$
f has a global min of $-1/2$ at $x = 3\pi/4$

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QUIZ 17 ♠

MATH 200
November 7, 2022

1. (10 points) Find the global extrema of the function $f(x) = x^3 - 75x + 10$ on $(0, \infty)$.

$$f'(x) = 3x^2 - 75 = 0$$
$$3x^2 = 75$$
$$x^2 = 25$$
$$x = \pm\sqrt{25} = \pm 5$$

The critical points are $x = 5$ and $x = -5$, but only $x = 5$ is in the interval $(0, \infty)$

$$f''(x) = 6x$$

$f''(5) = 6 \cdot 5 = 30 > 0$ So f has a local min at $x = 5$

However, 5 is the only critical point, so This is a global minimum

$f(x)$ has a global minimum at $x = 5$
$f(x)$ has no global maximum

2. (10 points) Find the global extrema of the function $f(x) = \sqrt[3]{x}^4 + 4\sqrt[3]{x}$ on $[-8, 8]$.

$$f(x) = x^{4/3} + 4x^{1/3}$$

$$f'(x) = \frac{4}{3}x^{\frac{1}{3}} + \frac{4}{3}x^{-2/3} = \frac{4}{3}\left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}^2}\right) = \frac{4}{3} \cdot \frac{x+1}{\sqrt[3]{x}^2}$$

The critical points are $\boxed{x = -1 \text{ and } x = 0}$ because $f'(-1) = 0$ and $f'(0)$ is not defined. Both of these critical points are in the interval $[-8, 8]$.

$$f(-8) = \sqrt[3]{-8}^4 + 4\sqrt[3]{-8} = (-2)^4 - 8 = 8$$
$$f(0) = \sqrt[3]{0}^4 + 4\sqrt[3]{0} = 0 + 0 = 0$$
$f(-1) = \sqrt[3]{-1}^4 + 4\sqrt[3]{-1} = 1 - 4 = -3$ ← global min
$f(8) = \sqrt[3]{8}^4 + 4\sqrt[3]{8} = 16 + 8 = 24$ ← global max