Name: Richard

QUIZ 17

MATH 200
October 24, 2024

1. Find the global extrema of $f(x) = \tan(x) - 2x$ on the interval $\left[0, \frac{\pi}{4}\right]$.

$$f'(x) = \sec^2(x) - 2 = 0$$
$$\sec^2(x) = 2$$
$$\sec(x) = \pm\sqrt{2}$$
$$\cos(x) = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

$x = \frac{\pi}{4}$ is the only critical point in $\left[0, \frac{\pi}{4}\right]$ and it happens to be an endpoint!

$f(0) = \tan(0) - 2 \cdot 0 = 0 \leftarrow$ max

$f\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) - 2 \cdot \frac{\pi}{4} = 1 - \frac{\pi}{2} < 0 \leftarrow$ min.

Ans. $\boxed{\begin{array}{l} f \text{ has a global max of } 0 \text{ at } x = 0 \\ f \text{ has a global min of } 1-\frac{\pi}{2} \text{ at } x = \frac{\pi}{4} \end{array}}$

2. Find the global extrema of $f(x) = \frac{x^2}{2} + \frac{8}{x}$ on the interval $(1, 4)$.

$$f'(x) = x - \frac{8}{x^2} = 0$$
$$x = \frac{8}{x^2}$$
$$x^3 = 8$$

$x = \sqrt[3]{8}$

$x = 2$
This is the only critical point in The interval

$$f''(x) = 1 + \frac{16}{x^3}$$

$f''(2) = 1 + \frac{16}{2^3} = 3 > 0$, so by 2^{nd} derivative test there is a local minimum at $x = 2$

Therefore $\boxed{f \text{ has a global minimum of } f(2) = 6 \text{ at } x = 2. \text{ No global maximum}}$

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$$f'(x) = x - \frac{8}{x^2} = 0$$
$$x = \frac{8}{x^2}$$
$$x^3 = 8 \rightarrow x = \sqrt[3]{8}$$

$x = 2$
This is the only critical point in $[1,4]$

$$f(1) = \frac{1^2}{2} + \frac{8}{1} = \frac{1}{2} + \frac{16}{2} = \frac{17}{2}$$

$$f(2) = \frac{2^2}{2} + \frac{8}{2} = 2 + 4 = 6 \leftarrow \text{min}$$

$$f(4) = \frac{4^2}{2} + \frac{8}{4} = 8 + 2 = 10 \leftarrow \text{max}$$

Ans: f has a global max of 10 at $x=4$
f has a global min of 6 at $x=2$

2. Find the global extrema of $f(x) = \tan(x) - 2x$ on the interval $\left(0, \frac{\pi}{2}\right)$.

$$f'(x) = \sec^2(x) - 2 = 0$$
$$\sec^2(x) = 2$$
$$\sec(x) = \pm\sqrt{2}$$
$$\cos(x) = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

$\rightarrow x = \frac{\pi}{4}$ is the only critical point in $(0, \pi/2)$ because $f'(\frac{\pi}{4}) = 0$

$$f''(x) = 2\sec(x)\sec(x)\tan(x) = 2\sec^2(x)\tan(x)$$

$f''(\frac{\pi}{4}) = 2\sqrt{2}^2 \cdot 1 = 4 > 0$ so by 2nd derivative test, f has a local min. at $x = \pi/4$, so this is a global min.

Ans: f has a global minimum of $f(\pi/4) = 1 - \frac{\pi}{2}$ at $x = \frac{\pi}{4}$. No global max