Name: Richard

QUIZ 17

MATH 200

October 23, 2025

This quiz concerns the function $f(x) = 3x^4 - 4x^3 - 12x^2$.

1. Find the critical points of f.

$$f'(x) = 12x^3 - 12x^2 - 24x = 0$$
$$12x(x^2 - x - 2) = 0$$
$$12x(x+1)(x-2) = 0$$

$\boxed{0 \quad -1 \quad 2}$ ← critical points

2. Find the global extrema of $f(x)$ on the interval $[-1, 1]$.

The only critical points in the interval are 0 & -1, and -1 happens to also be an endpoint.

$$f(-1) = 3(-1)^4 - 4(-1)^3 - 12(-1)^2 = 3 + 4 - 12 = -5$$
$$f(0) = 3(0)^4 - 4(0)^3 - 12(0)^2 = 0 - 0 - 0 = 0 \quad \leftarrow \text{MAX}$$
$$f(1) = 3(1)^4 - 4(1)^3 - 12(1^2) = 3 - 4 - 12 = -13 \quad \leftarrow \text{MIN}$$

$\boxed{\begin{array}{l}\text{The global maximum is } 0 \text{ at } x = 0 \\ \text{The global minimum is } -13 \text{ at } x = 1\end{array}}$

3. Find the global extrema of $f(x)$ on the interval $(1, 4)$.

There is only one critical point, $x = 2$, in this interval. Let's check it with the 2nd derivative test:

$$f''(x) = 36x^2 - 24x - 24$$
$$f''(2) = 36 \cdot 2^2 - 24 \cdot 2 - 24$$
$$= 144 - 48 - 24 > 0$$

This there is a local minimum at $x = 2$.

The global minimum is $f(2) = -8$ at $x = 2$

Name: Richard

QUIZ 17

MATH 200
October 23, 2025

This quiz concerns the function $f(x) = 3x^4 - 16x^3 + 18x^2$.

1. Find the critical points of f.

$$f'(x) = 12x^3 - 48x^2 + 36x = 0$$
$$12x(x^2 - 4 + 3) = 0$$
$$12x(x-1)(x-3)$$

$\boxed{0 \quad 1 \quad 3}$ ← critical points.

2. Find the global extrema of $f(x)$ on the interval $[-1, 1]$.

The only critical points in the interval are 0 and 1, and 1 happens to be an endpoint.

$$f(-1) = 3(-1)^4 - 16(-1)^3 + 18(-1)^2 = 3 + 16 + 18 = 37 \leftarrow \text{max}$$
$$f(0) = 3(0)^4 - 16(0)^3 + 18(0)^2 = 0 + 0 + 0 = 0 \leftarrow \text{min}$$
$$f(1) = 3(1)^4 - 16(1)^3 + 18(1)^2 = 3 - 16 + 18 = 5$$

The global maximum is $f(-1) = 37$ at $x = -1$
The global minimum is $f(0) = 0$ at $x = 0$

3. Find the global extrema of $f(x)$ on the interval $(0, 2)$.

There is only one critical point, $x=1$ in This interval
Let's test it with the 2nd derivative test.

$$f''(x) = 36x^2 - 96x + 36$$
$$f''(1) = 36 \cdot 1^2 - 96 \cdot 1 + 36 = 72 - 96 < 0$$

Thus there is a local maximum at $x = 1$.

The global maximum is $f(1) = 5$ at $x = 1$
No global minimum