Name: ______________________________ **Quiz 17** **MATH 200**
March 31, 2022

1. (10 points) Find the global extrema of the function $f(x) = x - 2\sqrt{x}$ on the closed interval $[0, 9]$.

   Notice that $f(x) = x - 2x^{1/2}$, so $f'(x) = 1 - 2 \cdot \frac{1}{2}x^{-1/2} = 1 - \frac{1}{\sqrt{x}}$.
   To find the critical points, solve $f'(x) = 0$.

$$\begin{aligned} 1 - \frac{1}{\sqrt{x}} &= 0 \\ 1 &= \frac{1}{\sqrt{x}} \\ \sqrt{x} &= 1 \\ \sqrt{x}^2 &= 1^2 \\ x &= 1 \end{aligned}$$

   Notice that the only critical point $x = 1$ **does** happen to be in the interval $[0, 4]$.

   $f(0) = 0 - 2\sqrt{0} = 0$
   $f(1) = 1 - 2\sqrt{1} = -1$
   $f(9) = 9 - 2\sqrt{9} = 3$

   Conclusion:
   The global minimum is -1, and it happens at $x = 1$.
   The global maximum is 3, and it happens at $x = 9$.

2. (10 points) Find the global extrema of the function $f(x) = \sin(x) - \frac{x}{2}$ on the open interval $\left(0, \frac{\pi}{2}\right)$.

   First find the critical points in the interval: $f'(x) = \cos(x) - \frac{1}{2}$

   Set this equal to zero: $\cos(x) - \frac{1}{2} = 0$

   We get $\cos(x) = \frac{1}{2}$, so the critical point is $x = \frac{\pi}{3}$.

   (This is the only value of x in $\left(0, \frac{\pi}{2}\right)$ for which $\cos(x) = \frac{1}{2}$.)

   As there is only one critical point in the interval, we just need to check if it gives a local max or min.

   For this we will use the second derivative test:

   $f''(x) = -\sin(x)$

   $f''\left(\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} < 0.$ Therefore f has a local maximum at $x = \frac{\pi}{3}$.

   Answer: $f(x) = \sin(x) - \frac{x}{2}$ has a global max of $f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ at $x = \frac{\pi}{3}$. No global min.

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1. (10 points) Find the global extrema of the function $f(x) = 8\sqrt{x} - x$ on the closed interval $[0, 25]$.

   Notice that $f(x) = 8x^{1/2} - x$, so $f'(x) = 8 \cdot \frac{1}{2}x^{-1/2} - 1 = \frac{4}{\sqrt{x}} - 1$.

   To find the critical points, solve $f'(x) = 0$.

$$\begin{aligned} \frac{4}{\sqrt{x}} - 1 &= 0 \\ \frac{4}{\sqrt{x}} &= 1 \\ 4 &= \sqrt{x} \\ 4^2 &= \sqrt{x}^2 \\ 16 &= x \end{aligned}$$

   Notice that the only critical point $x = 16$ **does** happen to be in the interval $[0, 25]$.

   $f(0) = 8\sqrt{0} - 0 = 0$
   $f(16) = 8\sqrt{16} - 16 = 16$
   $f(25) = 8\sqrt{25} - 25 = 15$

   Conclusion:
   The global minimum is 0, and it happens at $x = 0$.
   The global maximum is 16, and it happens at $x = 16$.

2. (10 points) Find the global extrema of the function $f(x) = \frac{x}{2} + \cos(x)$ on the open interval $\left(0, \frac{\pi}{2}\right)$.

   First find the critical points in the interval: $f'(x) = \frac{1}{2} - \sin(x)$

   Set this equal to zero: $\frac{1}{2} - \sin(x) = 0$

   We get $\sin(x) = \frac{1}{2}$, so the critical point is $x = \frac{\pi}{6}$.

   (This is the only value of x in $\left(0, \frac{\pi}{2}\right)$ for which $\sin(x) = \frac{1}{2}$.)

   As there is only one critical point the interval, we just need to check if it gives a local max or min.

   For this we will use the second derivative test:

   $f''(x) = -\cos(x)$

   $f''\left(\frac{\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} < 0.$ Therefore f has a local maximum at $x = \frac{\pi}{6}$.

   Answer: $f(x) = \frac{x}{2} + \cos(x)$ has a global max of $f''\left(\frac{\pi}{6}\right) = \frac{\pi}{12} + \frac{\sqrt{3}}{2}$ at $x = \frac{\pi}{6}$. No global min.