Name: Richard

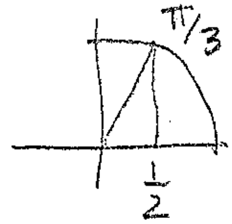
Quiz 17 ♡

MATH 200
March 26, 2026

1. Find the global extrema of the function $f(x) = \sin(x) - \frac{x}{2}$ on the closed interval $\left[0, \frac{\pi}{2}\right]$.

$$f'(x) = \cos(x) - \frac{1}{2} = 0$$

$$\cos(x) = \frac{1}{2}$$



Critical point: $x = \frac{\pi}{3}$

$$f(0) = \sin(0) - \frac{0}{2} = 0 - 0 = 0$$

$$f\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) - \frac{\pi/3}{2} = \frac{\sqrt{3}}{2} - \frac{\pi}{6} = \frac{3\sqrt{3} - \pi}{6} = \frac{3 \cdot 1.7 - 3.1}{6} = \frac{2}{6} = \frac{1}{3} = 0.3$$

$$f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) - \frac{\pi/2}{2} = 1 - \frac{\pi}{4} = \frac{4-\pi}{4} \approx \frac{4 - 3.1}{4} = \frac{0.9}{4} \approx 0.22$$

f has a global min of $f(0) = 0$ at $x = 0$
f has a global max of $f\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}-\pi}{6}$ at $x = \pi/3$

2. Find the global extrema of the function $f(x) = x - 2\sqrt{x}$ on the open interval $(0, 9)$.

$$f(x) = x - 2x^{1/2}$$

$$f'(x) = 1 - 2 \cdot \frac{1}{2} x^{-1/2} = 1 - \frac{1}{\sqrt{x}} = 0$$

Only one critical point!

$$1 = \frac{1}{\sqrt{x}}$$

Note that f'(0) is not defined, so x=0 is a critical point. However, we will ignore it because it is not in the interval (0,9).

$$\sqrt{x} = 1$$

$$x = 1$$

critical point

$$f''(x) = \frac{1}{2} x^{-3/2} = \frac{1}{2\sqrt{x}^3}$$

$$f''(x) = \frac{1}{2\sqrt{1}} = \frac{1}{2} > 0,$$ so local min. at $x = 1$

f has a global min of $f(1) = -1$ at $x = 1$
No global max.

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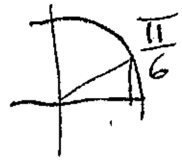
1. Find the global extrema of the function $f(x) = \frac{x}{2} + \cos(x)$ on the closed interval $\left[0, \frac{\pi}{2}\right]$.

$$f'(x) = \frac{1}{2} - \sin(x) = 0$$

$$\sin(x) = \frac{1}{2}$$



$$x = \frac{\pi}{6} \longleftarrow \text{critical point}$$

$$f(0) = \frac{0}{2} + \cos(0) = 1$$

$$f\left(\frac{\pi}{6}\right) = \frac{\pi/6}{2} + \cos\left(\frac{\pi}{6}\right) = \frac{\pi}{12} + \frac{\sqrt{3}}{2} = \frac{\pi + 6\sqrt{3}}{12} \approx \frac{3.1 + 6 \cdot 1.7}{12} = \frac{13.3}{2} > 1$$

$$f\left(\frac{\pi}{2}\right) = \frac{\pi/2}{2} + \cos\left(\frac{\pi}{2}\right) = \frac{\pi}{4} \approx \frac{3.14}{4} \approx 0.78$$

f has a global max of $f\left(\frac{\pi}{6}\right) = \frac{\pi + 6\sqrt{3}}{12}$ at $x = \frac{\pi}{6}$

f has a global min of $f\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$ at $x = \frac{\pi}{2}$

2. Find the global extrema of the function $f(x) = 8\sqrt{x} - x$ on the open interval $(0, 25)$.

$$f(x) = 8x^{\frac{1}{2}} - x$$

$$f'(x) = 8 \cdot \frac{1}{2} x^{-\frac{1}{2}} - 1 = \frac{4}{\sqrt{x}} - 1 = 0$$

$$\frac{4}{\sqrt{x}} = 1$$

$$4 = \sqrt{x}$$

$$\boxed{x = 16} \longleftarrow \text{critical point}$$

Only one critical point on the open interval!

Note that f'(0) is not defined, so x=0 is a critical point. However, we will ignore it because it is not in the interval (0,25).

$$f''(x) = -2x^{-\frac{3}{2}} = \frac{-2}{\sqrt{x}^3}$$

$$f''(16) = -\frac{2}{\sqrt{16}^3} = \frac{-2}{4^3} < 0 \longleftarrow \text{so local max at } x = 16$$

f has a global max of $f(16) = 16$ at $x = 16$
No global min.

Name: ______________________________ QUIZ 17 $\diamondsuit$ MATH 200

March 26, 2026

1. (10 points) Find the global extrema of the function $f(x) = x - 2\sqrt{x}$ on the closed interval $[0, 9]$.

   Notice that $f(x) = x - 2x^{1/2}$, so $f'(x) = 1 - 2 \cdot \frac{1}{2}x^{-1/2} = 1 - \frac{1}{\sqrt{x}}$.
   Because $f'(0)$ is undefined, $x = 0$ is a critical point. To find the other critical points, solve $f'(x) = 0$.

$$\begin{aligned} 1 - \frac{1}{\sqrt{x}} &= 0 \\ 1 &= \frac{1}{\sqrt{x}} \\ \sqrt{x} &= 1 \\ \sqrt{x}^2 &= 1^2 \\ x &= 1 \end{aligned}$$

   Notice that both critical points $x = 0, 1$ are in the interval $[0, 9]$, and 0 happens to be an endpoint.

   $f(0) = 0 - 2\sqrt{0} = 0$
   $f(1) = 1 - 2\sqrt{1} = -1$
   $f(9) = 9 - 2\sqrt{9} = 3$

   Conclusion:
   The global minimum is -1, and it happens at $x = 1$.
   The global maximum is 3, and it happens at $x = 9$.

2. (10 points) Find the global extrema of the function $f(x) = \sin(x) - \frac{x}{2}$ on the open interval $\left(0, \frac{\pi}{2}\right)$.

   First find the critical points in the interval: $f'(x) = \cos(x) - \frac{1}{2}$

   Set this equal to zero: $\cos(x) - \frac{1}{2} = 0$

   We get $\cos(x) = \frac{1}{2}$, so the critical point is $x = \frac{\pi}{3}$.

   (This is the only value of x in $\left(0, \frac{\pi}{2}\right)$ for which $\cos(x) = \frac{1}{2}$.)

   As there is only one critical point in the interval, we just need to check if it gives a local max or min.

   For this we will use the second derivative test:

   $f''(x) = -\sin(x)$

   $f''\left(\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} < 0.$ Therefore f has a local maximum at $x = \frac{\pi}{3}$.

   Answer: $f(x) = \sin(x) - \frac{x}{2}$ has a global max of $f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ at $x = \frac{\pi}{3}$. No global min.

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Quiz 17 ♠

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March 26, 2026

1. (10 points) Find the global extrema of the function $f(x) = 8\sqrt{x} - x$ on the closed interval $[0, 25]$.

Notice that $f(x) = 8x^{1/2} - x$, so $f'(x) = 8 \cdot \frac{1}{2}x^{-1/2} - 1 = \frac{4}{\sqrt{x}} - 1$.

Because $f'(0)$ is undefined, $x = 0$ is a critical point. To find the other critical points, solve $f'(x) = 0$.

$$
\begin{aligned}
\frac{4}{\sqrt{x}} - 1 &= 0 \\
\frac{4}{\sqrt{x}} &= 1 \\
4 &= \sqrt{x} \\
4^2 &= \sqrt{x}^2 \\
16 &= x
\end{aligned}
$$

Notice that both critical points $x = 0, 16$ are in the interval $[0, 25]$, and 0 happens to be an endpoint.

$f(0) = 8\sqrt{0} - 0 = 0$
$f(16) = 8\sqrt{16} - 16 = 16$
$f(25) = 8\sqrt{25} - 25 = 15$

Conclusion:
The global minimum is 0, and it happens at $x = 0$.
The global maximum is 16, and it happens at $x = 16$.

2. (10 points) Find the global extrema of the function $f(x) = \frac{x}{2} + \cos(x)$ on the open interval $\left(0, \frac{\pi}{2}\right)$.

First find the critical points in the interval: $f'(x) = \frac{1}{2} - \sin(x)$

Set this equal to zero: $\frac{1}{2} - \sin(x) = 0$

We get $\sin(x) = \frac{1}{2}$, so the critical point is $x = \frac{\pi}{6}$.

(This is the only value of x in $\left(0, \frac{\pi}{2}\right)$ for which $\sin(x) = \frac{1}{2}$.)

As there is only one critical point the interval, we just need to check if it gives a local max or min.

For this we will use the second derivative test:

$f''(x) = -\cos(x)$

$f''\left(\frac{\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} < 0.$ Therefore f has a local maximum at $x = \frac{\pi}{6}$.

Answer: $f(x) = \frac{x}{2} + \cos(x)$ has a global max of $f''\left(\frac{\pi}{6}\right) = \frac{\pi}{12} + \frac{\sqrt{3}}{2}$ at $x = \frac{\pi}{6}$. No global min.