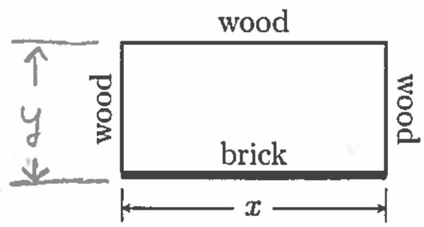
Name: Richard

QUIZ 18 ♡

MATH 200
November 10, 2021

1. A rectangular region of 600 square feet needs to be enclosed by a fence. The south side of the region will be bounded by a brick wall, and the fencing on the remaining three sides will be made of wood. The brick wall is \$10 per foot, and the wood wall costs \$5 per foot. Find the length x of the brick wall that results in the lowest cost of materials.

Let y be width of the rectangle, as shown here. →



Cost of materials = (cost of brick) + (cost of wood)

$= 10x + 5(y + x + y)$ dollars

$= 15x + 10y$

There is a constraint of area $= 600 = xy$, so $\boxed{y = \frac{600}{x}}$

Thus cost of materials is $15x + 10 \cdot \frac{600}{x}$

So we need to find the x that gives a global minimum of $C(x) = 15x + \frac{6000}{x}$ on $(0, \infty)$

Interval is $(0,\infty)$ because $x>0$, but otherwise could have any value, as $y = 600/x$

$C'(x) = 15 - \frac{6000}{x^2} = 0$

$15x^2 = 6000$

$x^2 = 400$

$x = \sqrt{400} = 20$ ← critical point

Note: $C''(x) = \frac{12000}{x^3}$ so $C''(20) = \frac{12000}{20^3} > 0$ and $C(x)$ has a local minimum at $x = 20$. As 20 is the only critical point, this is a global min.

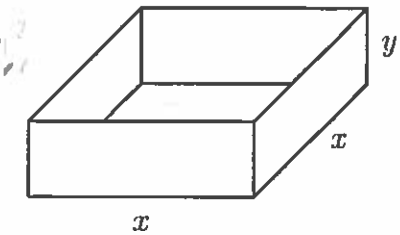
$\boxed{\text{Answer: Use dimensions } x = 20' \text{ and } y = \frac{600}{20} = 30'}$

Name: Richard

Quiz 18 ♣

MATH 200
November 10, 2021

1. A cardboard box with a square base and open top is to have a volume of 4 cubic meters. Find the dimensions that result in a box that uses the least cardboard.



Surface area of base: x^2 square meters,

Surface area of each side: xy sq. meters.

Total surface area: $S = x^2 + 4xy$

Constraint: Volume $= xxy = 4$ so $y = \frac{4}{x^2}$

Thus $S = x^2 + 4x \cdot \frac{4}{x^2} = x^2 + \frac{16}{x}$

Therefore we seek the x that gives a global minimum of $S(x) = x^2 + \frac{16}{x}$ on the interval $(0, \infty)$

Interval is $(0, \infty)$ because $0 < x$ but otherwise x could have any value, as $y = 4/x^2$

$$S'(x) = 2x - \frac{16}{x^2} = 0$$
$$2x = \frac{16}{x^2}$$
$$2x^3 = 16$$
$$x^3 = 8$$
$$x = \sqrt[3]{8} = 2 \leftarrow \text{critical point}$$

Notice that $S''(x) = 2 + \frac{32}{x^3}$, so $S''(2) = 2 + \frac{32}{2^3} > 0$ so $S(x)$ has a local minimum at $x = 2$ by the second derivative test. Because 2 is the <u>only</u> critical point, this is a <u>global minimum</u>.

<u>Answer</u> Use dimensions $x = 2$ meters and $y = \frac{4}{2^2} = 1$ meter

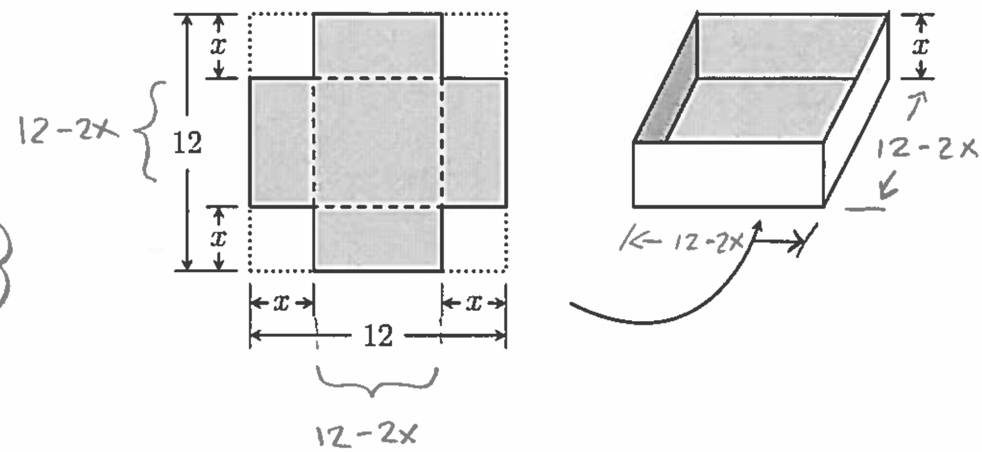
Name: Richard

QUIZ 18 ◊

MATH 200
November 10, 2021

1. An open-top box is made from a 12 by 12 inch piece of cardboard by cutting a square from each corner, and folding up. What should x be to maximize the volume of the box?

Note: x cannot be greater than half of 12, so $0 < x < 6$



Note that the box has length $12-2x$, width $12-2x$ and height x. Therefore its volume is

$$V(x) = (12-2x)(12-2x)\,x = (144 - 48x + 4x^2)\,x = 144x - 48x^2 + 4x^3.$$

Thus we want to find the global maximum of $V(x) = 144x - 48x^2 + 4x^3$ on $(0, 6)$

$$V'(x) = 144 - 96x + 12x^2 = 12(x^2 - 8x + 12) = 12(x-2)(x-6) = 0$$

$x = 2$ $\quad$ $x = 6$

Critical points. Note that only $x = 2$ is in the interval

$V''(x) = -96 + 24x$

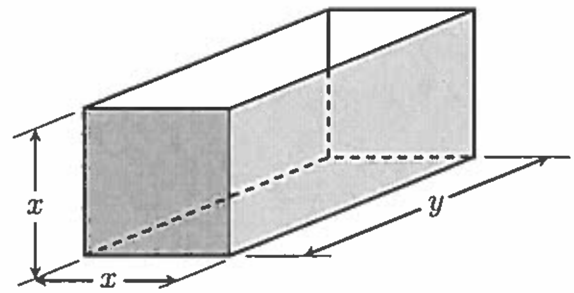
$V''(2) = -96 + 24 \cdot 2 = -48 < 0$, so local max at $x = 2$ (by 2^{nd} derivative test). Since 2 is the only critical point in $(0,6)$, This gives a global max. $\boxed{\text{Answer } x = 2''}$

Name: Richard

QUIZ 18 ♠

MATH 200
November 10, 2021

1. A metal box with two square ends and an open top is to contain a volume of 36 cubic inches. What dimensions x and y will minimize the total area of the metal surface?



Total surface area is

$$A = 2x^2 + 2xy + xy$$

two ends, two sides, bottom

$$A = 2x^2 + 3xy$$

constraint:

Volume $= 36 = x \cdot x \cdot y$

so $y = \frac{36}{x^2}$

$$A = 2x^2 + 3x\frac{36}{x^2}$$

$$= 2x^2 + \frac{108}{x}$$

Thus we seek the x that gives a global minimum of $A(x) = 2x^2 + \frac{108}{x}$ on the interval $(0, \infty)$.

Interval is $(0, \infty)$ because $x > 0$, but could otherwise be any value, as $y = 36/x^2$

$$A'(x) = 4x - \frac{108}{x^2} = 0$$

$$4x = \frac{108}{x^2}$$

$$x^3 = 27$$

$$x = \sqrt[3]{27} = 3$$ ← critical point

$A''(x) = 4 + \frac{216}{x^3}$ so $A''(3) = 4 + \frac{216}{3^2} > 0$ and so $A(x)$ has a local minimum at $x = 3$. Since 3 is the only critical point this is a global minimum

Answer: $\boxed{\text{Use dimensions } x = 3 \text{ and } y = \frac{36}{3^2} = 4}$