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QUIZ 18 ♢

MATH 200
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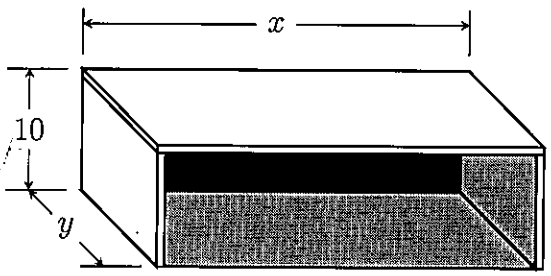
1. A 10-foot-high shed with flat roof and three walls is to be constructed, as illustrated below. It is required that the shed have 800 square feet of floor space. Materials for the roof cost \$2 per square foot. Materials for the walls cost \$1 per square foot. Materials for the floor cost \$3 per square foot. What dimensions x and y will minimize the total cost of materials?



Cost = roof + floor + walls

$= 2xy + 3xy + 1(10y + 10y + 10x)$

$= 5xy + 20y + 10x$

Constraint

floor space $= 800 = xy$

$y = \frac{800}{x}$

$= 5x \cdot \frac{800}{x} + 20\frac{800}{x} + 10x$

$= 4000 + \frac{16000}{x} + 10x$

Cost $= C(x) = 4000 + \frac{16000}{x} + 10x$ ← Find global minimum on $(0, \infty)$

$C'(x) = -\frac{16000}{x^2} + 10 = 0$

$10 = \frac{16000}{x^2}$

$10x^2 = 16000$

$x^2 = 1600$

$x = \sqrt{1600} = 40$ ← critical point

Is there a max or min at $x = 40$? To see, use the second derivative test,

$C''(x) = \frac{32000}{x^3}$ and $C''(40) = \frac{32000}{40^3} > 0$ so minimum!

Answer: Use $x = 40'$ and $y = \frac{800}{40} = 20'$