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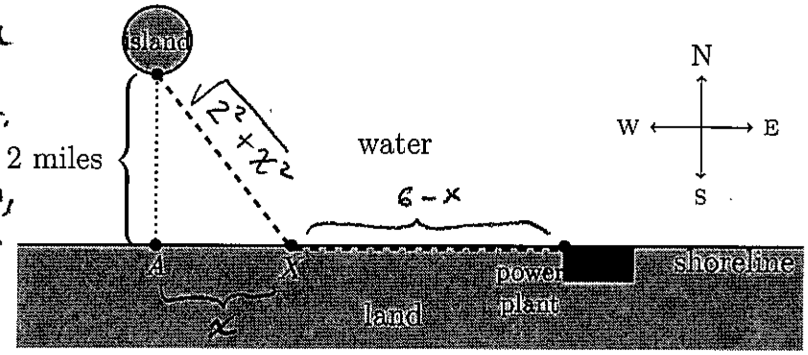
QUIZ 18

MATH 200

October 31, 2023

1. An island is 2 miles from the nearest point A on a straight shoreline. Point A is 6 miles from a power plant. A utility company plans to lay electrical cable underwater from the island to the shore, and then along the shore to the power station. (As shown by the dashed line, below.) It costs \$2,000 per mile to lay the cable underwater, and \$1,000 per mile to run it along the shore. At what point X should the underwater cable meet the shore to minimize the cost of the project?



Let x be the distance between points A and x. We need to find the x that minimizes cost.

By the Pythagorean Theorem, there are $\sqrt{2^2+x^2} = \sqrt{4+x^2}$ miles of underwater cable. Also, there are $6-x$ miles of shoreline cable. Therefore

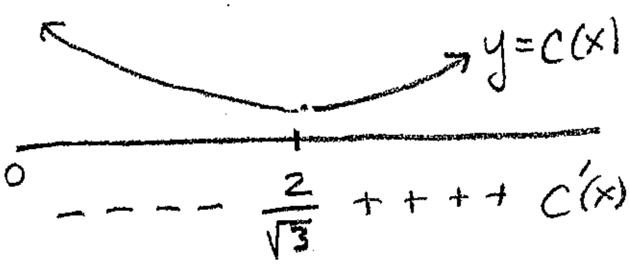
$$\text{Cost} = C(x) = 2000\sqrt{4+x^2} + 1000(6-x)$$

$$\boxed{C(x) = 2000\sqrt{4+x^2} + 6000 - 1000x}$$

We need to find the x that gives a global minimum of $C(x)$ on $[0, 6]$

$$C'(x) = \frac{2000 \cdot 2x}{2\sqrt{4+x^2}} + 0 - 1000$$

$$C'(x) = \frac{2000x}{\sqrt{4+x^2}} - 1000 = 0$$



$$\frac{2000x}{\sqrt{4+x^2}} = 1000$$

$$\frac{2x}{\sqrt{4+x^2}} = 1$$

$$2x = \sqrt{4+x^2}$$

$$4x^2 = 4 + x^2$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

critical point! $\boxed{x = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}}$

Test point $x=0$ $\quad C'(0) = -1000 < 0$

Test point $x=2$ $\quad C'(2) = \frac{2000 \cdot 2}{\sqrt{4+2^2}} - 1000 = \frac{2000}{\sqrt{2}} - 1000 > 0$

By 1st derivative test, $C(x)$ has a local min at $x = \frac{2}{\sqrt{3}}$. That's the only critical point, so it's a global min.

Ans: $\boxed{\text{For minimum cost, put } x = \frac{2}{\sqrt{3}} \text{ miles}}$