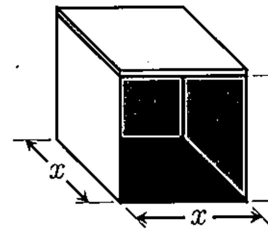
Name: Richard

QUIZ 18

MATH 200
October 29, 2024

1. You need to build a shed with an open front and square base (as illustrated), and containing a volume of 10,000 cubic feet.
   The cost of materials are:
   Roof: \$10 per square foot;
   Floor: \$5 per square foot.
   Walls: \$8 per square foot;

   Find the dimensions x and y that will minimize the total cost of materials.



$$\text{Cost} = \text{roof} + \text{floor} + \text{walls}$$
$$= 10x^2 + 5x^2 + 3 \cdot 8 \cdot xy$$
$$= 10x^2 + 5x^2 + 24x\,\frac{10{,}000}{x^2}$$

Constraint:
Volume $= x^2 y = 10{,}000$
$\Rightarrow \boxed{y = \dfrac{10{,}000}{x^2}}$

$$C(x) = 15x^2 + \frac{240{,}000}{x}$$

We need to find x that gives a global minimum of this cost function on the interval $(0, \infty)$.

$$C'(x) = 30x - \frac{240000}{x^2} = 0$$
$$\frac{x^2}{30}\left(30x - \frac{2400000}{x^2}\right) = 0 \cdot \frac{x^2}{30}$$
$$x^3 - 8000 = 0$$
$$x = \sqrt[3]{8000} = \boxed{20}$$

critical pt

$C''(x) = x + \dfrac{480000}{x^3}$ and $C''(20) > 0$ so there is a local (hence global) minimum at $x = 20$ feet.

Then $y = \dfrac{10000}{20^2} = \dfrac{10000}{400} = \dfrac{100}{4} = 25$ feet

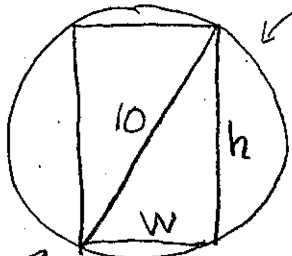
---

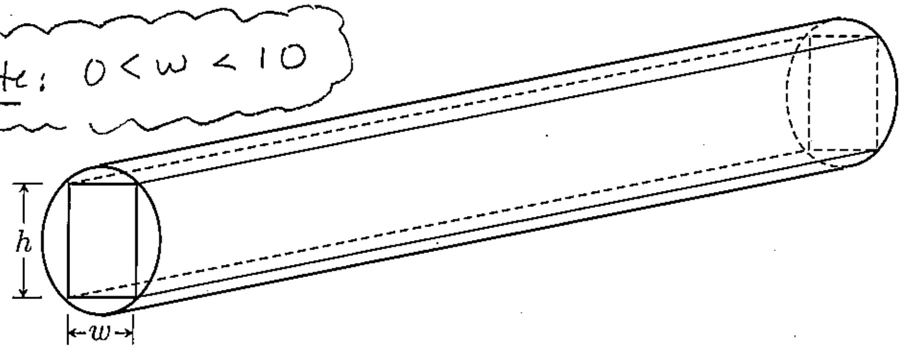
Answer: $\boxed{\text{Dimensions should be } x = 20,\ y = 25}$

Name: Richard — QUIZ 18 — MATH 200, October 29, 2024

1. The strength of a rectangular beam is directly proportional to the product of its width and the square of its height. Find the dimension of the strongest beam that can be cut from a cylindrical log of diameter 10 inches.

Note: $0 < w < 10$





Side view. By Pythagorean Thm, $w^2 + h^2 = 10^2$

Hence $\boxed{h^2 = 100 - w^2}$ or $\boxed{h = \sqrt{100 - w^2}}$

$$\text{Strength} = wh^2 = w(100 - w^2) = 100w - w^3$$

Therefore we need to find the w that gives a global maximum of strength $S(w) = 100w - w^3$ on the interval $(0, 10)$

$$S'(w) = 100 - 3w^2 = 0$$

$$w^2 = \frac{100}{3}$$

$$w = \sqrt{\frac{100}{3}} = \frac{10}{\sqrt{3}}$$ ← critical pt.

$$S''(w) = -6w$$

Note: $S''\left(\frac{10}{\sqrt{3}}\right) = -6 \cdot \frac{10}{\sqrt{3}} = -\frac{60}{\sqrt{3}} < 0$

So there is a local (hence global) maximum at $w = \frac{10}{\sqrt{3}}$. Then $h = \sqrt{100 - \left(\frac{10}{\sqrt{3}}\right)^2} = \sqrt{100 - \frac{100}{3}}$

$= \sqrt{\frac{200}{3}} = 10\sqrt{\frac{2}{3}}$ Answer: $\boxed{w = \frac{10}{\sqrt{3}} \quad h = 10\sqrt{\frac{2}{3}}}$