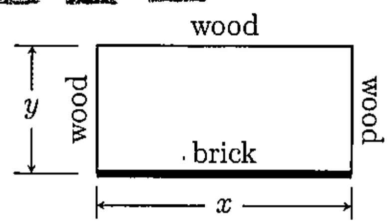
Name: Richard

Quiz 18

MATH 200
October 28, 2025

1. Imagine that you have a budget of \$300 for materials to enclose a rectangular region with a fence. The south side of the rectangle will be bounded by a brick wall, and the fencing on the remaining three sides will be made of wood. The brick wall is \$10 per foot, and the wood wall is \$5 per foot. Given the above, find the dimensions x and y that enclose the greatest possible area.



We are asked to maximize area $= xy$.

To turn this into a function of x, use this constraint:

$$\text{COST} = \begin{pmatrix}\text{cost of}\\ \text{South}\\ \text{side}\end{pmatrix} + \begin{pmatrix}\text{cost of}\\ \text{east}\\ \text{side}\end{pmatrix} + \begin{pmatrix}\text{Cost of}\\ \text{north}\\ \text{side}\end{pmatrix} + \begin{pmatrix}\text{Cost of}\\ \text{west}\\ \text{side}\end{pmatrix}$$

$$300 = 10x + 5y + 5x + 5y$$

$$300 = 15x + 10y$$

$$10y = 300 - 15x$$

$$y = \frac{1}{10}(300 - 15x) \Rightarrow \left\{ y = 30 - \frac{3}{2}x \right\}$$

Now we have $\text{AREA} = xy = x\left(30 - \frac{3}{2}x\right) = 30x - \frac{3}{2}x^2$,

that is, area $= \left\{ A(x) = 30x - \frac{3}{2}x^2 \right\}$

The problem becomes the following

Find the global maximum of $A(x) = 30x - \frac{3}{2}x^2$ on $(0, \infty)$.

$$A'(x) = 30 - 3x = 0$$

$$30 = 3x$$

$$x = 10$$

only one critical point

$A''(x) = -3 < 0$ so this is a global maximum at $x = 10$.

When $x = 10$, $y = 30 - \frac{3}{2} \cdot 10 = 30 - 15 = 15$

Answer

Greatest area if

$X = 10$

$y = 15$

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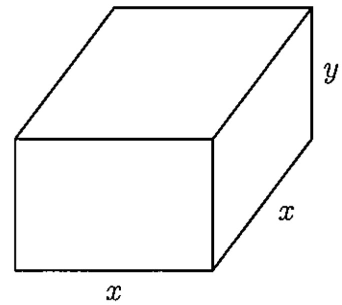
1. Imagine you need to design a tank with a square base that holds 10,000 cubic feet of water. The metal top costs \$6 per square foot, and the concrete sides and bottom cost \$4 per square foot. What dimensions x and y yield the lowest cost of materials?



$$\text{Cost} = \begin{pmatrix}\text{cost of}\\ \text{top}\end{pmatrix} + \begin{pmatrix}\text{cost of}\\ \text{bottom}\end{pmatrix} + 4\begin{pmatrix}\text{cost of}\\ \text{one}\\ \text{side}\end{pmatrix}$$

$$= 6x^2 + 4x^2 + 4\cdot 4xy$$

$$= 10x^2 + 16xy$$

$$= 10x^2 = 16x\frac{10000}{x^2}$$

$$C(x) = 10x^2 + \frac{160000}{x}$$

Constraint:

$$10000 = x\cdot x\cdot y$$

$$y = \frac{10000}{x^2}$$

Minimize this on $(0,\infty)$

$$C'(x) = 20x - \frac{160000}{x^2} = 0$$

$$20x^3 = 160000$$

$$x^3 = 8000$$

$$x = \sqrt[3]{8000} = 20$$

only one critical point

$C''(x) = 20 + \frac{32000}{x^3} \Rightarrow C''(20) > 0$ so there is a local (hence global) minimum of cost $C(x)$ when $x = 20$. Then the constraint yields $y = \frac{10000}{20^2} = \frac{10000}{400} = \frac{100}{4} = 25$.

ANSWER $\boxed{\text{Cost is minimized when } \begin{cases} x = 20 \\ y = 25 \end{cases}}$