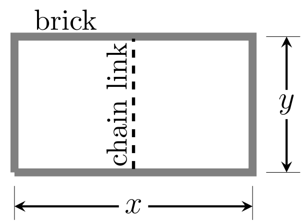
Name: ______________________________

QUIZ 18

MATH 200
April 5, 2022

1. A total area of 2000 square feet is to be enclosed by two pens, as illustrated. The outside walls will be made of brick, and the inner dividing wall is chain link. The brick wall costs \$10 per foot, and the chain link costs \$5 per foot. Find the dimensions x and y that minimize the cost of construction.



Strategy: We want to minimize cost, so let's build a function for cost.

$$\begin{aligned} \text{Cost} &= \text{cost of btick} + \text{cost of chain link} \\ &= 10(2x+2y) \quad + \quad 5y \\ &= 20x \quad + \quad 25y. \end{aligned}$$

The constraint is: Area $= x \cdot y = 2000$ square feet. Therefore $\boxed{y = \dfrac{2000}{x}}$. Inserting this above,

$$\begin{aligned} \text{Cost} &= 20x \quad + \quad 25 \cdot \frac{2000}{x} \\ &= 20x + \frac{50000}{x}. \end{aligned}$$

Therefore we seek the x that gives a global minimum of the function $C(x) = 20x + \dfrac{50000}{x}$ on $(0, \infty)$.

To find the critical points, solve:

$$\begin{aligned} C'(x) &= 0 \\ 20 - \frac{50000}{x^2} &= 0 \\ 20 &= \frac{50000}{x^2} \\ x^2 &= 2500 \\ x &= \sqrt{2500} = 50 \end{aligned}$$

Sot there is only one critical point, $x = 50$.

Does this give a global max or min?

To find out, let's use the second derivative test: $C''(x) = \dfrac{100000}{x^3}$

Note that $C''(50) = \dfrac{100000}{50^3} > 0$ is positive, so $C(x)$ has a **local minimum** at $x = 50$.

But because 50 is the only critical point in the interval, there is a **global minimum there**.

When $x = 50$, the constraint gives $y = \dfrac{2000}{50} = 40$

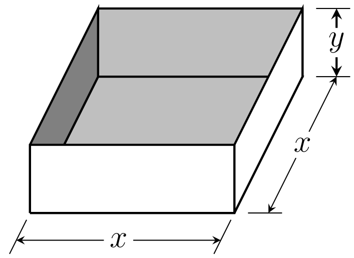
Answer:

$\boxed{\text{Dimensions } x = 50 \text{ feet and } y = 40 \text{ feet minimize cost.}}$

Name: ______________________ Quiz 18 MATH 200

April 5, 2022

1. You need to build a square concrete-lined pool having a box-like shape and a volume of 500 cubic feet.



In order to minimize costs you want the (concrete-lined) surface area to be as small as possible. What dimensions x and y result in a volume of 500 cubic feet, but with the smallest possible surface area?

Strategy: We want to minimize surface area, so let's build a function for surface area.

$$\begin{aligned}\text{Surface area} &= 4 \text{ sides} + \text{bottom}\\ &= 4 \cdot xy \;\;+\;\; x^2.\end{aligned}$$

The constraint is: Volume $= x \cdot x \cdot y = 500$ cubic feet. Therefore $\boxed{y = \dfrac{500}{x^2}}$. Inserting this above,

$$\begin{aligned}\text{Surface area} &= 4 \cdot x \cdot \frac{500}{x^2} \;\;+\;\; x^2\\ &= \frac{2000}{x} + x^2.\end{aligned}$$

Therefore we seek the x that gives a global minimum of the function $S(x) = \dfrac{2000}{x} + x^2$ on $(0, \infty)$.

To find the critical points, solve:

$$\begin{aligned}S'(x) &= 0\\ -\frac{2000}{x^2} + 2x &= 0\\ 2x &= \frac{2000}{x^2}\\ x^3 &= 1000\\ x &= \sqrt[3]{1000} = 10\end{aligned}$$

Sot there is only one critical point, $x = 10$.

Does this give a global max or min?

To find out, let's use the second derivative test: $S''(x) = \dfrac{4000}{x^3} + 2$

Note that $S''(10) = \dfrac{4000}{10^3} + 2 = 6 > 0$ is positive, so $S(x)$ has a **local minimum** at $x = 10$.

But because 10 is the only critical point in the interval, there is a **global minimum there**.

When $x = 10$, the constraint gives $y = \dfrac{500}{10^2} = 5$

Answer:

Dimensions $x = 10$ feet and $y = 5$ feet minimize surface area and therefore also minimize cost.