Name: Richard

QUIZ 19 ♡

MATH 200
November 15, 2021

1. Consider $f(x) = x^2 + 2x$ on $[0, 5]$. Find *all* numbers c in $(0, 5)$ guaranteed by the mean value theorem.

$$\frac{f(5)-f(0)}{5-0} = \frac{(5^2+2\cdot 5)-(0^2+2\cdot 0)}{5} = \frac{35}{5} = 7$$

Also $f'(x) = 2x+2$.

We seek c for which $f'(c) = \frac{f(5)-f(0)}{5-0}$

$$2c+2 = 7$$
$$2c = 5$$
$$c = 5/2$$

Answer: $\boxed{c = \frac{5}{2}}$ is the number for which $f'(c) = \frac{f(5)-f(0)}{5-0}$

2. Suppose $f(x)$ is a function, and $f(50) = -20$ and $f'(50) = 7$. Based on this information, find the linear approximation $L(x)$ for $f(x)$ at 50. Then use it to find an approximate value of $f(51)$.

$$L(x) = f(a) + f'(a)(x-a)$$
$$= f(50) + f'(50)(x-50)$$
$$= -20 + 7(x-50)$$
$$= -20 + 7x - 350$$

$\boxed{L(x) = 7x - 370}$ or $\boxed{L(x) = -20 + 7(x-50)}$

$$f(51) \approx L(51) = -20 + 7(51-50) = \boxed{-13}$$

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QUIZ 19 ♣

MATH 200
November 15, 2021

1. Consider $f(x) = 4 - x^2$ on $[1, 2]$. Find *all* numbers c in $(1, 2)$ guaranteed by the mean value theorem.

We seek the numbers c in [1,2] for which

$$f'(c) = \frac{f(2) - f(1)}{2-1}$$

$$-2c = \frac{(4-2^2) - (4-1^2)}{2-1}$$

$$-2c = \frac{0-3}{1}$$

$$\boxed{c = 3/2}$$

Answer $c = 3/2$ in $[1,2]$ satisfies $f'(c) = \frac{f(2)-f(1)}{2-1}$

2. Suppose $f(x)$ is a function, and $f(11) = 10$ and $f'(11) = -2$. Based on this information, find the linear approximation $L(x)$ for $f(x)$ at 11. Then use it to approximate value of $f(10)$.

$$L(x) = f(a) + f'(a)(x-a)$$

$$L(x) = f(11) + f'(11)(x-11)$$

$$L(x) = 10 + (-2)(x-11)$$

$$\boxed{L(x) = 10 - 2(x-11)}$$

or $\boxed{L(x) = -2x + 32}$

$$\boxed{f(10) \approx L(10) = -2 \cdot 10 + 32 = 12}$$

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Quiz 19 ◊

MATH 200
November 15, 2021

1. Consider $f(x) = x^2 + 2x - 3$ on $[-3, 0]$. Find *all* numbers c in $(-3, 0)$ guaranteed by the mean value theorem.

We need to solve $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$$2c + 2 = \frac{(0^2 + 2\cdot 0 - 3) - ((-3)^2 + 2(-3) - 3)}{0 - (-3)}$$

$$2c + 2 = \frac{-3 - 0}{3}$$

$$2c + 2 = -1$$

$$2c = -3$$

$$c = -3/2$$

Answer $\boxed{c = -3/2}$ is the number for which $f'(c) = \dfrac{f(0) - f(-3)}{0 - (-3)}$

2. Suppose $f(x)$ is a function, and $f(90) = -10$ and $f'(90) = 7$. Based on this information, find the linear approximation $L(x)$ for $f(x)$ at 90. Then use it to find an approximate value of $f(91)$.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = f(90) + f'(90)(x - 90)$$

$$\boxed{L(x) = -10 + 7(x - 90)}$$

or $\boxed{L(x) = 7x - 640}$

$$f(91) \approx L(91) = -10 + 7(91 - 90) = -10 + 7 = \boxed{-3}$$

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QUIZ 19 ♠

MATH 200
November 15, 2021

1. Consider $f(x) = x^3 - 2x + 4$ on $[0, 2]$. Find *all* numbers c in $(0, 2)$ guaranteed by the mean value theorem.

We need to solve $f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$

$$3c^2 - 2 = \frac{(2^3 - 2\cdot 2 + 4) - (0^3 - 2\cdot 0 + 4)}{2}$$

$$3c^2 - 2 = \frac{8 - 4}{2}$$

$$3c^2 - 2 = 2$$

$$3c^2 = 4$$

$$c^2 = \frac{4}{3} \longrightarrow c = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$$

Note: $-\frac{2}{\sqrt{3}}$ is not in $(0,2)$

Answer: $\boxed{c = \frac{2}{\sqrt{3}}}$ is the number guaranteed by MVT.

2. Suppose $f(x)$ is a function, and $f(50) = -20$ and $f'(50) = 7$. Based on this information, find the linear approximation $L(x)$ for $f(x)$ at 50. Then use it to find an approximate value of $f(51)$.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = f(50) + f'(50)(x - 50)$$

$$\boxed{L(x) = -20 + 7(x - 50)}$$

or $\boxed{L(x) = 7x - 370}$

$$f(51) \approx L(51) = -20 + 7(51 - 50) = -20 + 7 = \boxed{-13}$$