Name: Richard

QUIZ 19 ♡

MATH 200
November 14, 2022

1. This problem concerns the function $f(x) = \ln|x|$. $f'(x) = \frac{1}{x}$

   (a) (2 pts.) Does the mean value theorem hold for f on the interval $[-1, 1]$? Why or why not?

   <u>No</u> $f(x) = \ln|x|$ is <u>not</u> continuous at $x = 0 \in [-1, 1]$

   (b) (2 pts.) Does the mean value theorem hold for f on the interval $[1, e]$? Why or why not?

   <u>YES</u> $f(x) = \ln|x|$ is continuous on $[1, e]$ and differentiable on $(1, e)$

   (c) (2 pts.) Does the mean value theorem hold for f on the interval $[0, 1]$? Why or why not?

   <u>NO</u> $f(x)$ is not continuous at $0 \in [0, 1]$.

   (d) (4 pts.) If the mean value theorem holds for one of the above intervals, find all numbers $x = c$ in the interval that are guaranteed by the theorem.

   We seek an x for which $f'(x) = \frac{f(e) - f(1)}{e - 1}$

   Use interval $[1, e]$

   $$\frac{1}{x} = \frac{\ln(e) - \ln(1)}{e - 1}$$

   $$\frac{1}{x} = \frac{1 - 0}{e - 1}$$

   $$\boxed{x = e - 1}$$

2. In this problem $f(x)$ is a function for which $f(10) = -7$ and $f'(10) = 2$.

   (a) (6 pts.) Find the linear approximation for $f(x)$ at 10.
   Put your answer in the form $L(x) = mx + b$.

   $$\begin{aligned} L(x) &= f(a) + f'(a)(x - a) \\ &= f(10) + f'(10)(x - 10) \\ &= -7 + 2(x - 10) \\ &= -7 + 2x - 20 \\ &= 2x - 27 \end{aligned}$$

   $$\boxed{L(x) = 2x - 27}$$

   (b) (4 pts.) Use your answer from part (a) to find the approximate value of $f(11)$.

   $$f(11) \approx L(11) = 2 \cdot 11 - 27 = 22 - 27 = \boxed{-5}$$

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QUIZ 19 ♣

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1. This problem concerns the function $f(x) = \dfrac{1}{x}$.

   (a) (2 pts.) Does the mean value theorem hold for f on the interval $[-1, 1]$? Why or why not?

   No $f(x)=\frac{1}{x}$ is neither continuous nor differentiable at $0 \in [-1,0]$

   (b) (2 pts.) Does the mean value theorem hold for f on the interval $[0, 1]$? Why or why not?

   No $f(x) = \frac{1}{x}$ is not continuous at $0 \in [0,1]$.

   (c) (2 pts.) Does the mean value theorem hold for f on the interval $[1, 2]$? Why or why not?

   Yes $f(x) = \frac{1}{x}$ is continuous on $[1,2]$ and differentiable on $(1,2)$

   (d) (4 pts.) If the mean value theorem holds for one of the above intervals, find all numbers $x = c$ in the interval that are guaranteed by the theorem.

   We seek all x for which $f'(x) = \dfrac{f(2)-f(1)}{2-1}$

   Using interval $[1,2]$

$$-\frac{1}{x^2} = \frac{\frac{1}{2} - \frac{1}{1}}{2-1}$$

$$-\frac{1}{x^2} = \frac{-\frac{1}{2}}{1}$$

$$\frac{1}{x^2} = \frac{1}{2}$$

$$x^2 = 2 \longrightarrow \boxed{x=\sqrt{2}}$$

   Reject $-\sqrt{2}$ since it's not in $[1,2]$

2. In this problem $f(x)$ is a function for which $f(5) = 4$ and $f'(5) = -2$.

   (a) (6 pts.) Find the linear approximation for $f(x)$ at 5.
   Put your answer in the form $L(x) = ax + b$.

$$L(x) = f(5) + f'(5)(x-5)$$
$$= 4 + (-2)(x-5)$$
$$= 4 - 2x + 10$$
$$\boxed{L(x) = -2x + 14}$$

   (b) (4 pts.) Use your answer from part (a) to find the approximate value of $f(5.5)$.

$$f(5.5) \approx L(5.5) = -2\cdot 5.5 + 14 = -11 + 14 = \boxed{3}$$

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QUIZ 19 ◇

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1. This problem concerns the function $f(x) = \frac{1}{x-1}$. $= (x-1)^{-1}$ $\quad f'(x) = -(x-1)^{-2}(1) = \frac{-1}{(x-1)^2}$

   (a) (3 pts.) Does the mean value theorem hold for f on the interval $[0,3]$? Why or why not?

   No $f(1) = \frac{1}{1-1} = \frac{1}{0}$ is not defined, so f is not continuous on $[0,3]$.

   (b) (3 pts.) Does the mean value theorem hold for f on the interval $[2,3]$? Why or why not?

   Yes because f is continuous on $[2,3]$ and differentiable on $(2,3)$.

   (c) (4 pts.) If the mean value theorem holds for one of the above intervals, find all numbers $x = c$ in the interval that are guaranteed by the theorem.

   We seek all numbers x in $(2,3)$ for which

   $$f'(x) = \frac{f(3) - f(2)}{3-2}$$

   $$\frac{-1}{(x-1)^2} = \frac{\frac{1}{3-1} - \frac{1}{2-1}}{3-2}$$

   $$\frac{-1}{(x-1)^2} = \frac{\frac{1}{2} - \frac{1}{1}}{1}$$

   $$\frac{-1}{(x-1)^2} = -\frac{1}{2}$$

   $$(x-1)^2 = 2$$

   $$x - 1 = \pm\sqrt{2}$$

   $$x = 1 \pm \sqrt{2}$$

   Reject $1-\sqrt{2}$ because that's not in $(2,3)$.

   $\boxed{\text{Answer: } x = 1+\sqrt{2}}$

2. In this problem $f(x)$ is a function for which $f(15) = 2$ and $f'(15) = -3$.

   (a) (6 pts.) Find the linear approximation for $f(x)$ at 15.
   Put your answer in the form $L(x) = mx + b$.

   $$\begin{aligned} L(x) &= f(a) + f'(a)(x-a) \\ &= f(15) + f'(15)(x-15) \\ &= 2 - 3(x-15) \\ &= 2 - 3x + 45 \\ &= -3x + 47 \end{aligned}$$

   $\boxed{L(x) = -3x + 47}$

   (b) (4 pts.) Use your answer from part (a) to find the approximate value of $f(16)$.

   $f(16) \approx L(16) = -3 \cdot 16 + 47 = -48 + 47 = \boxed{-1}$

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QUIZ 19 ♠

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1. This problem concerns the function $f(x) = \sqrt[3]{x}^2$. $= x^{2/3}$ $\quad f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

(a) (3 pts.) Does the mean value theorem hold for f on the interval $[-1, 1]$? Why or why not?

No because f is not differentiable at $0 \in [-1,1]$

(b) (3 pts.) Does the mean value theorem hold for f on the interval $[0, 8]$? Why or why not?

Yes because f is continuous on $[0,8]$ and differentiable on $(0,8)$.

(c) (4 pts.) If the mean value theorem holds for one of the above intervals, find all numbers $x = c$ in the interval that are guaranteed by the theorem.

We seek an $x \in [0,8]$ for which

$$f'(x) = \frac{f(8) - f(0)}{8 - 0}$$

$$\frac{2}{3\sqrt[3]{x}} = \frac{\sqrt[3]{8}^2 - \sqrt[3]{0}^2}{8}$$

$$\frac{2}{3\sqrt[3]{x}} = \frac{2^2 - 0}{8}$$

$$\frac{2}{3\sqrt[3]{x}} = \frac{1}{2} \longrightarrow$$

$$3\sqrt[3]{x} = 4$$

$$\sqrt[3]{x} = \frac{4}{3}$$

$$x = \left(\frac{4}{3}\right)^3$$

$$\boxed{x = \frac{64}{27}}$$

2. In this problem $f(x)$ is a function for which $f(100) = 3$ and $f'(100) = -5$.

(a) (6 pts.) Find the linear approximation for $f(x)$ at 100.
Put your answer in the form $L(x) = mx + b$.

$$\begin{aligned} L(x) &= f(a) + f'(a)(x-a) \\ &= f(100) + f'(100)(x-100) \\ &= 3 - 5(x-100) \\ &= 3 - 5x + 500 \\ &= -5x + 503 \end{aligned}$$

$$\boxed{L(x) = -5x + 503}$$

(b) (4 pts.) Use your answer from part (a) to find the approximate value of $f(101)$.

$$f(101) \approx L(101) = -5(101) + 503 = -505 + 503 = \boxed{-2}$$