

1. (10 pts.) State the Mean Value Theorem.

If a function $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) then there exists a number c in (a, b) for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

2. In this problem $f(x)$ is a function for which $f(4) = 3$ and $f'(4) = -2$.

- (a) (6 pts.) Find the linear approximation for $f(x)$ at 4.

Put your answer in the form $L(x) = mx + b$.

$$\begin{aligned} L(x) &= f(a) + f'(a)(x-a) \\ &= f(4) + f'(4)(x-4) \\ &= 3 + (-2)(x-4) \\ &= 3 - 2x + 8 \end{aligned}$$

$$\boxed{L(x) = -2x + 11}$$

- (b) (4 pts.) Use your answer from part (a) to find the approximate value of $f(3.5)$.

$$\begin{aligned} f(3.5) &\approx L(3.5) = -2(3.5) + 11 \\ &= -7 + 11 = \boxed{4} \end{aligned}$$

1. (10 pts.) State the Mean Value Theorem.

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$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

2. In this problem $f(x)$ is a function for which $f(3) = 5$ and $f'(3) = -2$.

- (a) (6 pts.) Find the linear approximation for $f(x)$ at 3.

Put your answer in the form $L(x) = mx + b$.

$$\begin{aligned} L(x) &= f(a) + f'(a)(x - a) \\ &= f(3) + f'(3)(x - 3) \\ &= 5 + (-2)(x - 3) \\ &= 5 - 2x + 6 \end{aligned}$$

$$L(x) = -2x + 11$$

- (b) (4 pts.) Use your answer from part (a) to find the approximate value of $f(2.5)$.

$$\begin{aligned} f(2.5) &\approx L(2.5) = -2(2.5) + 11 \\ &= -5 + 11 = \boxed{6} \end{aligned}$$