Name: Richard

QUIZ 19

MATH 200
October 30, 2025

1. (10 pts.) State the Mean Value Theorem.

If a function $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c in (a, b) for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

2. This problem concerns the function $f(x) = 2\ln(x) + 3$.

(a) (6 pts.) Find the linear approximation for $f(x)$ at $x = 1$.
Put your answer in the form $L(x) = mx + b$.

$$f'(x) = \frac{2}{x}$$

$$f'(1) = \frac{2}{1} = 2$$

$$f(1) = 2\ln(1) + 3 = 0 + 3 = 3$$

$$L(x) = f(1) + f'(1)(x-1) = 3 + 2(x-1) = 3 + 2x - 2 = 2x + 1$$

$$\boxed{L(x) = 2x + 1}$$

(b) (4 pts.) Use your answer from part (a) to find the approximate value of $f(1.25)$.

$$f(1.25) \approx L(1.25) = 2(1.25) + 1 = 2.5 + 1 = \boxed{3.5}$$

Name: Richand

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1. (10 pts.) State the Mean Value Theorem.

If a function $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) then there is a number c in (a, b) for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

2. This problem concerns the function $f(x) = 7 + e^{2x-4}$.

(a) (6 pts.) Find the linear approximation for $f(x)$ at $x = 2$.
Put your answer in the form $L(x) = mx + b$.

$f'(x) = 2e^{2x-4}$

$f'(2) = 2e^{2\cdot 2-4} = 2\cdot e^0 = 2$

$f(2) = 7 + e^{2\cdot 2-4} = 7 + e^0 = 7+1 = 8$

$$\begin{aligned} L(x) &= f(2) + f'(2)(x-2) \\ &= 8 + 2(x-2) \\ &= 8 + 2x - 4 \\ &= 2x + 4 \end{aligned}$$

$\boxed{L(x) = 2x+4}$

(b) (4 pts.) Use your answer from part (a) to find the approximate value of $f(2.25)$.

$$\begin{aligned} f(2.25) &\approx 2\cdot 2.25 + 4 \\ &= 4.5 + 4 \\ &= \boxed{8.5} \end{aligned}$$