Name: Richard

QUIZ 19 ♡

MATH 200

April 2, 2026

a b

1. Consider $f(x) = x + \sin(x)$ on $[0, 2\pi]$. Find *all* numbers c in $(0, 2\pi)$ guaranteed by the mean value theorem.

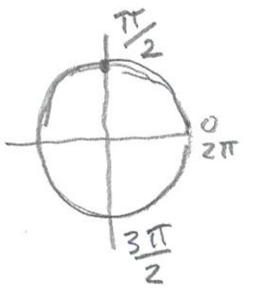
Such a number c obeys $f'(c) = \dfrac{f(2\pi) - f(0)}{2\pi - 0}$

$$1 + \cos(c) = \frac{(2\pi + \sin(2\pi)) - (0 + \sin(0))}{2\pi}$$

$$1 + \cos(c) = \frac{(2\pi + 0) - (0 + 0)}{2\pi}$$

$$1 + \cos(c) = \frac{2\pi}{2\pi}$$

$$\cos(c) = 0$$



$$\Downarrow$$

$$\boxed{c = \frac{\pi}{2} \text{ and } c = \frac{3\pi}{2}}$$

2. Find the linear approximation $L(x)$ for the function $f(x) = x + \dfrac{3}{x}$ at $x = 3$.

$$f'(x) = 1 - \frac{3}{x^2}$$

$$\begin{aligned}
L(x) &= f(3) + f'(3)(x-3) \\
&= 3 + \frac{3}{3} + \left(1 - \frac{3}{3^2}\right)(x-3) \\
&= 4 + \left(1 - \frac{1}{3}\right)(x-3) \\
&= 4 + \frac{2}{3}(x-3) \\
&= 4 + \frac{2}{3}x - 2 \\
&= \frac{2}{3}x + 2
\end{aligned}$$

$$\boxed{L(x) = \frac{2}{3}x + 2}$$

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QUIZ 19 ♣

MATH 200
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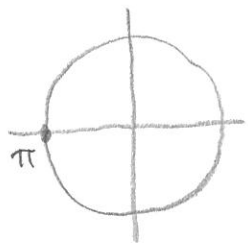
1. Consider $f(x) = 2 + \cos(x)$ on $[0, 2\pi]$. Find *all* numbers c in $(0, 2\pi)$ guaranteed by the mean value theorem.

a b

Such a number c obeys $f'(c) = \dfrac{f(2\pi) - f(0)}{2\pi - 0}$

$$-\sin(c) = \frac{(2+\cos(2\pi)) - (2+\cos(0))}{2\pi}$$



$$-\sin(c) = \frac{(2+1)-(2+1)}{2\pi}$$

$$-\sin(c) = \frac{0}{2\pi}$$

$$\sin(c) = 0$$

$$\Downarrow$$

Answer $\boxed{c = \pi}$

2. Find the linear approximation $L(x)$ for the function $f(x) = 2e^x - x$ at $x = \ln(3)$.

$$f'(x) = 2e^x - 1$$

$$L(x) = f(\ln(3)) + f'(\ln(3))(x - \ln(3))$$

$$= 2e^{\ln(3)} - \ln(3) + (2e^{\ln(3)} - 1)(x - \ln(3))$$

$$= 2\cdot 3 - \ln(3) + (2\cdot 3 - 1)(x - \ln(3))$$

$$= 6 - \ln(3) + 5(x - \ln(3))$$

$$= 6 - \ln(3) + 5x - 5\ln(3)$$

$$= 5x + 6 - 6\ln(3)$$

$$\boxed{L(x) = 5x + 6 - 6\ln(3)}$$