Name: Richard

QUIZ 21 ♡

MATH 200
November 28, 2022

1. Find using any appropriate method: $\lim_{x\to\infty} (e^x - 1)^{1/x} = \lim_{x\to\infty} e^{\ln((e^x-1)^{1/x})}$

form ∞^0

$$= \lim_{x\to\infty} e^{\frac{1}{x}\ln(e^x-1)} = \lim_{x\to\infty} e^{\frac{\ln(e^x-1)}{x}}$$

$$= e^{\lim_{x\to\infty}\frac{\ln(e^x-1)}{x}} = e^{\lim_{x\to\infty}\frac{\frac{e^x}{e^x-1}}{1}} = e^{\lim_{x\to\infty}\frac{e^x}{e^x-1}}$$

form $\frac{\infty}{\infty}$

form $\frac{\infty}{\infty}$

$$= e^{\lim_{x\to\infty}\frac{e^x}{e^x}} = e^1 = \boxed{e}$$

2. $\int (x^{-1} + 1 + x^1 + x^2)\, dx = \boxed{\ln|x| + x + \frac{x^2}{2} + \frac{x^3}{3} + C}$

3. $\int (\sqrt{x} + \sin(x))\, dx = \int\left(x^{\frac{1}{2}} + \sin(x)\right)dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \cos(x) + C$

$$= \frac{x^{3/2}}{3/2} - \cos(x) + C = \boxed{\frac{2}{3}\sqrt{x}^3 - \cos(x) + C}$$

4. $\int (e^x + e^2)\, dx = \boxed{e^x + xe^2 + C}$

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1. Find using any appropriate method: $\lim\limits_{x\to 0^+} (e^x - 1)^{1/x} = \boxed{0}$

form 0^{∞}
Not indeterminate!
L'Hopital does not apply.
Notice $0^{\infty} = 0$

2. $\int (x^6 + x^{1/2} + x + 2)\, dx = \frac{x^7}{7} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^2}{2} + 2x + C$

$= \boxed{\frac{x^7}{7} + \frac{2x^{3/2}}{3} + \frac{x^2}{2} + 2x + C}$

3. $\int \left(\frac{1}{x} + \frac{1}{x^2}\right) dx = \int\left(\frac{1}{x} + x^{-2}\right)dx = \ln|x| + \frac{x^{-2+1}}{-2+1} + C$

$= \ln|x| + \frac{x^{-1}}{-1} + C = \boxed{\ln|x| - \frac{1}{x} + C}$

4. $\int (2e^x + \sec^2(x))\, dx = \int 2e^x dx + \int \sec^2(x)\, dx$

$= 2\int e^x dx + \int \sec^2(x)\, dx$

$= \boxed{2e^x + \tan(x) + C}$

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1. Find using any appropriate method: $\lim_{x\to\infty} (e^x)^{1/(x+1)} = \lim_{x\to\infty} e^{\ln((e^x)^{1/(x+1)})}$

form ∞^0

$= \lim_{x\to\infty} e^{\frac{1}{x+1}\ln(e^x)} = \lim_{x\to\infty} e^{\frac{x}{x+1}}$ ← using $\ln(e^x) = x$

$= e^{\lim_{x\to\infty} \frac{x}{x+1}} = e^{\lim_{x\to\infty} \frac{1}{1+0}} = e^1 = \boxed{e}$

form $\frac{\infty}{\infty}$

2. $\int (3x^2 + 2 + 3x)\, dx = 3\frac{x^3}{3} + 2x + 3\frac{x^2}{2} + C$

$= \boxed{x^3 + 2x + \frac{3}{2}x^2 + C}$

3. $\int \left(e^x + \frac{1}{e}\right)\, dx = \boxed{e^x + \frac{x}{e} + C}$ ← $\frac{1}{e}$ is a constant

so $\int \frac{1}{e}\, dx = \frac{1}{e}x + C$

$= \frac{x}{e} + C$

4. $\int \frac{2}{1+x^2}\, dx = 2\int \frac{1}{1+x^2}\, dx = \boxed{2\tan^{-1}(x) + C}$

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1. Find using any appropriate method: $\lim_{x\to 0}(x^2+1)^{1/x} = \lim_{x\to 0} e^{\ln|(x^2+1)^{1/x}|}$

indeterminate form 1^∞

$$= \lim_{x\to 0} e^{\frac{1}{x}\ln|x^2+1|}$$

$$= \lim_{x\to 0} e^{\frac{\ln|x^2+1|}{x}} = e^{\lim_{x\to 0}\frac{\ln|x^2+1|}{x}}$$

form $\frac{0}{0}$

$$= e^{\lim_{x\to 0}\frac{\frac{2x}{x^2+1}}{1}} = e^{\lim_{x\to 0}\frac{2x}{x^2+1}} = e^0 = \boxed{1}$$

2. $\int (20x^4 - x^{-1} + x^{-2})\,dx = 20\frac{x^5}{5} - \ln|x| + \frac{x^{-2+1}}{-2+1} + C$

$\frac{1}{x^2}$

$$= 4x^5 - \ln|x| + \frac{x^{-1}}{-1} + C$$

$$= \boxed{4x^5 - \ln|x| - \frac{1}{x} + C}$$

3. $\int (e + e^x)\,dx = \boxed{xe + e^x + C}$

Note: e is a constant, so $\int e\,dx = ex + C$

4. $\int (\sqrt{x} + \cos(x))\,dx = \int\left(x^{\frac{1}{2}} + \cos(x)\right)dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \sin(x) + C$

$$= \frac{x^{3/2}}{3/2} + \sin(x) + C = \boxed{\frac{2}{3}\sqrt{x}^{\,3} + \sin(x) + C}$$