Name: Richard

Quiz 21 ♡

MATH 200
April 9, 2026

1. $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(\left(1+1/x\right)^x\right)} = \lim_{x\to\infty} e^{x\ln\left(1+\frac{1}{x}\right)}$

form 1^∞

$\displaystyle = e^{\lim\limits_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}}$

form $\frac{0}{0}$

$\displaystyle = e^{\lim\limits_{x\to\infty}\frac{\frac{-1/x^2}{1+1/x}}{\frac{-1}{x^2}}} = e^{\lim\limits_{x\to\infty}\frac{\cancel{-1/x^2}}{1+1/x}\cdot\frac{1}{\cancel{-1/x^2}}}$

$\displaystyle = e^{\lim\limits_{x\to\infty}\frac{1}{1+1/x}} = e^{\frac{1}{1+0}} = e^1 = \boxed{e}$

2. $\displaystyle\int\frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{-1/2+1}}{-1/2+1}+C = \frac{x^{1/2}}{1/2}+C$

$= \boxed{2\sqrt{x}+C}$

3. $\displaystyle\int\left(\frac{1}{x}-\sec^2(x)+\pi\right)dx = \boxed{\ln|x|-\tan(x)+\pi x+C}$

4. $\displaystyle\int\left(x^2+\frac{1}{x^2}\right)dx = \int\left(x^2+x^{-2}\right)dx = \frac{x^{2+1}}{2+1}+\frac{x^{-2+1}}{-2+1}+C$

$\displaystyle = \frac{x^3}{3}+\frac{x^{-1}}{-1}+C = \boxed{\frac{x^3}{3}-\frac{1}{x}+C}$

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1. $\lim\limits_{x\to 0+} (1+x)^{1/x} = \lim\limits_{x\to 0^+} e^{\ln((1+x)^{1/x})} = \lim\limits_{x\to 0^+} e^{\frac{1}{x}\ln(1+x)}$

form 0^∞

$= \lim\limits_{x\to 0^+} e^{\frac{\ln(1+x)}{x}} = e^{\lim\limits_{x\to 0^+}\frac{\ln(1+x)}{x}}$

form $\frac{0}{0}$

$= e^{\lim\limits_{x\to 0^+}\frac{\frac{1}{1+x}}{1}} = e^{\frac{1}{1+0}/1} = e^1 = \boxed{e}$

2. $\int \left(14x^6 - \frac{1}{x} + e^x\right) dx = 14\frac{x^7}{7} - \ln|x| + e^x + C$

$= \boxed{2x^7 - \ln|x| + e^x + C}$

3. $\int (5+\cos(x))\,dx = \boxed{5x + \sin(x) + C}$

4. $\int x\sqrt{x}\,dx = \int x^1 x^{\frac{1}{2}}\,dx = \int x^{1+\frac{1}{2}}\,dx = \int x^{3/2}\,dx$

$= \dfrac{x^{3/2+1}}{\frac{3}{2}+1} + C = \dfrac{x^{5/2}}{5/2} + C = \boxed{\frac{2}{5}\sqrt{x}^5 + C}$