Name: Richard

QUIZ 22 ♡

MATH 200

December 1, 2021

1. (6 points) $\int \frac{x^3+x^2+x}{x^2}\,dx = \int \left(\frac{x^3}{x^2} + \frac{x^2}{x^2} + \frac{x}{x^2}\right) dx$

$$= \int \left(x + 1 + \frac{1}{x}\right) dx = \boxed{\frac{x^2}{2} + x + \ln|x| + C}$$

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = \frac{1}{x} + 2x$ and $f(1) = 5$. Find $f(x)$.

$$f(x) = \int \left(\frac{1}{x} + 2x\right) dx = \ln|x| + 2\frac{1}{2}x^2 + C$$

Thus $f(x) = \ln|x| + x^2 + C$

Also $5 = f(1) = \ln|1| + 1^2 + C$

$5 = 0 + 1 + C$

$C = 4$

Answer: $\boxed{f(x) = \ln|x| + x^2 + 4}$

3. (7 points) A falling object has a velocity of $-32t - 16$ feet per second t seconds after it is dropped. It hits ground 10 seconds after being dropped. From what height was it dropped?

$V(t) = -32t - 16$

$S(t) = \int (-32t - 16)\,dt = -16t^2 - 16t + C$

Thus $S(t) = -16t^2 - 16t + C$

Know $0 = S(10) = -16 \cdot 10^2 - 16 \cdot 10 + C$

$0 = -1600 - 160 + C$

$C = 1760$

Consequently $\boxed{S(t) = -16t^2 - 16t + 1760}$

Object dropped from height $S(0) = \boxed{1760 \text{ feet}}$

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1. (6 points) $\int \frac{x-1}{x}\,dx = \int\left(\frac{x}{x}-\frac{1}{x}\right)dx = \int\left(1-\frac{1}{x}\right)dx$

$= \boxed{x - \ln|x| + C}$

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = e^x + 2x$ and $f(0) = 5$. Find $f(x)$.

$f(x) = \int(e^x + 2x)\,dx = e^x + x^2 + C$

So $f(x) = e^x + x^2 + C$

Also $5 = f(0) = e^0 + 0^2 + C$

$5 = 1 + C$

Thus $C = 4$ and $\boxed{f(x) = e^x + x^2 + 4}$

3. (7 points) An object moving on the number line has velocity $v(t) = 3t^2 + 4$ at time t seconds. It is at the point 2 on the number line the instant its acceleration is 12 units per second per second. Find the position function $s(t)$.

$v(t) = 3t^2 + 4$

$a(t) = v'(t) = 6t$ ← Acceleration is 12 when $t = 2$

The object is at point 2 when $t = 2$ (because $a(2) = 12$).

This means $\boxed{s(2) = 2}$

Also $s(t) = \int v(t)\,dt = \int(3t^2 + 4) = t^3 + 4t + C$

So $s(t) = t^3 + 4t + C$

To find C: $2 = s(2) = 2^3 + 4 \cdot 2 + C \Rightarrow C = 2 - 16 = -14$

Thus $\boxed{s(t) = t^3 + 4t - 14}$

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1. (6 points) $\int \frac{3x^2+5x}{x^2}\,dx = \int\left(\frac{3x^2}{x^2}+\frac{5x}{x^2}\right)dx = \int\left(3+5\cdot\frac{1}{x}\right)dx$

$$= \boxed{3x + 5\ln|x| + C}$$

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = 2x + \cos(x)$ and $f(\pi) = 0$. Find $f(x)$.

$f(x) = \int (2x + \cos(x))\,dx = 2\frac{x^2}{2} + \sin(x) + C$

so $f(x) = x^2 + \sin(x) + C$

Also, $0 = f(\pi) = \pi^2 + \sin(\pi) + C$

$0 = \pi^2 + 0 + C$

$C = -\pi^2$

$$\boxed{f(x) = x^2 + \sin(x) - \pi^2}$$

3. (7 points) Suppose an object moving on a line has velocity function $v(t) = 2t + 3$. Find its position function $s(t)$, given that you happen to know $s(2) = 8$.

$s(t) = \int (2t+3)\,dt = 2\frac{1}{2}t^2 + 3t + C$

i.e. $s(t) = t^2 + 3t + C$

Know $8 = s(2) = 2^2 + 3\cdot 2 + C$

$8 = 10 + C$

$C = -2$

Therefore $\boxed{s(t) = t^2 + 3t - 2}$

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1. (6 points) $\int \frac{x + xe^x}{x}\,dx = \int \frac{x(1+e^x)}{x} dx = \int (1+e^x)\,dx = \boxed{x + e^x + C}$

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = 3\sqrt{x} - 2$ and $f(4) = 7$. Find $f(x)$.

$$f(x) = \int (3\sqrt{x} - 2)\,dx = \int (3x^{1/2} - 2)\,dx = 3\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} - 2x + C$$

$$= 3\frac{1}{3/2}x^{3/2} - 2x + C = 2\sqrt{x}^3 - 2x + C$$

Therefore $\boxed{f(x) = 2\sqrt{x}^3 - 2x + C}$

To find C: $7 = f(4) = 2\sqrt{4}^3 - 2\cdot 4 + C$

$7 = 2\cdot 2^3 - 8 + C$

$C = -1$

Answer: $\boxed{f(x) = 2\sqrt{x}^3 - 2x - 1}$

3. (7 points) A ball, tossed straight up, has a constant acceleration of −32 feet per second per second. At time $t = 0$ its velocity is $v(0) = 10$ feet per second, and its position is $s(0) = 6$ feet. Find the position function $s(t)$.

$a(t) = -32$

$v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C$

So $\boxed{v(t) = -32t + C}$

To find C: $10 = v(0) = -32\cdot 0 + C$, so $\boxed{C = 10}$

Thus $\boxed{v(t) = -32t + 10}$

Now $s(t) = \int v(t)\,dt = \int (-32t + 10)\,dt = \boxed{-16t^2 + 10t + C.}$

i.e. $\boxed{s(t) = -16t^2 + 10t + C}$

To find C: $6 = s(0) = -16\cdot 0^2 + 10\cdot 0 + C \Rightarrow \boxed{C = 6}$ $\boxed{\text{Ans } s(t) = -16t^2 + 10t + 6}$