Name: Richard

QUIZ 22

MATH 200
November 16, 2023

1. (6 points) $\int \frac{4x^2-9}{2x+3}\,dx = \int \frac{(2x-3)\cancel{(2x+3)}}{\cancel{2x+3}}dx = \int (2x-3)\,dx$

$= 2\frac{x^2}{2} - 3x + C = \boxed{x^2 - 3x + c}$

2. (7 points) At the point $(x, f(x))$, the tangent to the graph of a function $y = f(x)$ has slope $m = 1 + \frac{1}{x^2}$. Also, the graph of $f(x)$ passes though the point $(3, 7)$. Find $f(x)$.

From the above we know $f'(x) = 1 + \frac{1}{x^2}$ and $f(3) = 7$.

Thus $f(x) = \int \left(1 + \frac{1}{x^2}\right)dx = \int (1 + x^{-2})\,dx = x - x^{-1} + c$

i.e $\boxed{f(x) = x - \frac{1}{x} + c}$

Also $7 = f(3) = 3 - \frac{1}{3} + c$, so $\boxed{c = 7 - 3 + \frac{1}{3} = 4 + \frac{1}{3} = \frac{13}{3}}$

Ans $\boxed{f(x) = x - \frac{1}{x} + \frac{13}{3}}$

3. (7 points) Given the velocity function, $v(t) = 2\sin(t) + 5t$ of an object moving along a line, find the position function with the initial condition $s(0) = b$. Your final answer should be in terms of b.

$S(t) = \int (2\sin(t) + 5t)\,dt = -2\cos(t) + 5\frac{t^2}{2} + C$

$b = S(0) = -2\cos(0) + 5\frac{0^2}{2} + C = -2 + C$

i.e. $C = b + 2$

$\therefore \boxed{S(t) = -2\cos(t) + \frac{5t^2}{2} + b + 2}$

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1. (6 points) $\int \frac{9x^2 - 16}{3x+4}\,dx = \int \frac{(3x-4)\cancel{(3x+4)}}{\cancel{(3x+4)}}\,dx = \int (3x-4)\,dx$

$= \boxed{3\frac{x^2}{2} - 4x + C}$

2. (7 points) At the point $(x, f(x))$, the tangent to the graph of a function $y = f(x)$ has slope $m = x + \frac{1}{x}$. Also, the graph of $f(x)$ passes though the point $(-e, 3)$. Find $f(x)$.

From the above we know $f'(x) = x + \frac{1}{x}$ and $f(-e) = 3$.

$f(x) = \int \left(x + \frac{1}{x}\right) dx = \boxed{\frac{x^2}{2} + \ln|x| + C}$

So $3 = f(-e) = \frac{(-e)^2}{2} + \ln|-e| + C = \frac{e^2}{2} + 1 + C$

i.e. $3 = \frac{e^2}{2} + 1 + C$, so $C = 2 - e^2/2$

Ans $\boxed{f(x) = \frac{x^2}{2} + \ln|x| + 2 - \frac{e^2}{2}}$

3. (7 points) Given the velocity function, $v(t) = e^t + 4$ of an object moving along a line, find the position function with the initial condition $s(0) = b$. Your final answer should be in terms of b.

$s(t) = \int (e^t + 4) = e^t + 4t + C$

$b = s(0) = e^0 + 4 \cdot 0 + C = 1 + C$

i.e. $C = b - 1$

$\boxed{s(t) = e^t + 4t + b - 1}$