Name: Richard

QUIZ 22

MATH 200

November 14, 2024

1. (6 points) $\int \frac{x^2 - 9x}{x^2}\,dx = \int\left(\frac{x^2}{x^2} - \frac{9x}{x^2}\right)dx = \int\left(1 - \frac{9}{x}\right)dx$

$$= \boxed{x - 9\ln|x| + C}$$

2. (7 points) The graph of a function $f(x)$ passes through the point $(2, 5)$, and the slope of the tangent line to the graph at any point $(x, f(x))$ is $m = 4x^3 + 2x + 1$. Find the function $f(x)$.

Know $f'(x) = 4x^3 + 2x + 1$,

so $f(x) = \int 4x^3 + 2x + 1\,dx = 4\frac{x^4}{4} + 2\frac{x^2}{2} + x + C$

i.e. $f(x) = x^4 + x^2 + x + C$

Know $5 = f(2) = 2^4 + 2^2 + 2 + C \Rightarrow 5 = 22 + C \Rightarrow \boxed{C = -17}$

Answer $\boxed{f(x) = x^4 + x^2 + x - 17}$

3. (7 points) A rocket lifting off from the surface of the moon has a constant acceleration of 4 meters per second per second. How high is the rocket 10 seconds after liftoff? (You may assume that its velocity is zero at the instant of liftoff.)

Know $a(t) = 4$, so velocity is $v(t) = \int a(t)\,dt = \int 4\,dt$ $= 4t + C$, that is $v(t) = 4t + C$. But $0 = v(0) = 4\cdot 0 + C$, which gives $C = 0$, so $\boxed{v(t) = 4t}$

Now, the height at time t is $s(t) = \int v(t)\,dt$ $= \int 4t\,dt = 2t^2 + C$, i.e. $s(t) = 2t^2 + C$. We know $0 = s(0) = 4\cdot 0^2 + C$, hence $C = 0$ and $\boxed{s(t) = 2t^2}$

ANS $s(10) = 2\cdot 10^2 = \boxed{200 \text{ meters}}$

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1. (6 points) $\int \frac{4x^2 - x}{x}\, dx = \int \frac{x(4x-1)}{\cancel{x}} dx = \int 4x - 1\, dx = 4\frac{x^2}{2} - x + C$

$= \boxed{2x^2 - x + C}$

2. (7 points) A rocket lifting off from the surface of the moon has a constant acceleration of 5 meters per second per second. How high is the rocket 10 seconds after liftoff? (You may assume that its velocity is zero at the instant of liftoff.)

$a(t) = 5$

$V(t) = \int a(t)\, dt = \int 5\, dt = 5t + C$

Know $0 = V(0) = 5 \cdot 0 + C$, so $C = 0$. $\boxed{V(t) = 5t}$

Height at time t is $S(t) = \int V(t)\, dt = \int 5t\, dt = 5\frac{t^2}{2} + C$

Know $0 = S(0) = \frac{5}{2}0^2 + C$, so $C = 0$.

Therefore $\boxed{S(t) = \frac{5}{2}t^2}$

Height at time $t = 10$ is $S(10) = \frac{5}{2}10^2 = \boxed{250 \text{ meters}}$

3. (7 points) The graph of a function $f(x)$ passes through the point $(2, 5)$, and the slope of the tangent line to the graph at any point $(x, f(x))$ is $m = 3x^2 + 4x + 1$. Find the function $f(x)$.

Know $f'(x) = 3x^2 + 4x + 1$,

so $f(x) = \int 3x^2 + 4x + 1\, dx = 3\frac{x^3}{3} + 4\frac{x^2}{2} + x + C$

i.e. $f(x) = x^3 + 2x^2 + x + C$

Know $5 = f(2) = 2^3 + 2 \cdot 2^2 + 2 + C \Rightarrow 5 = 18 + C \Rightarrow C = -13$

Answer $\boxed{f(x) = x^3 + 2x^2 + x - 13}$