Name: Richard

QUIZ 22

MATH 200
November 13, 2025

1. (6 points) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \boxed{\sin^{-1}(x) + C}$ because $D_x\left[\sin^{-1}(x)+C\right] = \frac{1}{\sqrt{1-x^2}}$

2. (7 points) The graph of a function $f(x)$ passes through the point $(2,5)$, and the tangent line to the graph at any point $(x, f(x))$ has slope $m = 4x^3 + 2x + 1$. Find the function $f(x)$.

Know $f'(x) = 4x^3 + 2x + 1$, therefore

$$f(x) = \int 4x^3 + 2x + 1\,dx = 4\frac{x^4}{4} + 2\frac{x^2}{2} + x + C$$

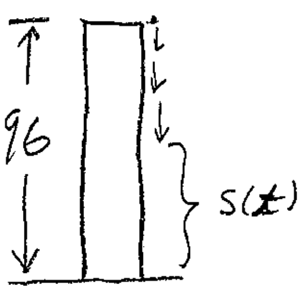
$$\boxed{f(x) = x^4 + x^2 + x + C}$$

Also, $5 = f(2) = 2^4 + 2^2 + 2 + C \implies 5 = 16 + 4 + 2 + C$

$$\implies C = 5 - 22 = -17$$

$$\boxed{f(x) = x^4 + x^2 + x - 17}$$

3. (7 points) A rock is dropped from the top of a 96 foot tall tower, and has a constant acceleration of -32 feet per second per second. Use calculus to find how long it takes the rock to fall to the ground. (You may assume that its velocity is zero the instant it's dropped.)



Say the rock is dropped at time $t=0$ and its height at time t is $s(t)$.

Velocity at time t is $v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C$. But we know $0 = v(0) = -32\cdot 0 + C$, so $C = 0$ and $\boxed{v(t) = -32t}$.

Then $s(t) = \int v(t)\,dt = \int -32t\,dt = -16t^2 + C$. But we know $96 = s(0) = -16\cdot 0^2 + C \implies C = 96 \implies \boxed{s(t) = 96 - 16t^2}$.

To find when rock strikes ground solve $s(t) = 0$, i.e. $96 - 16t^2 = 0 \implies 16t^2 = 96 \implies t^2 = 6 \implies t = \sqrt{6}$. $\boxed{\text{Ans } \sqrt{6} \text{ seconds}}$

Name: Richard

QUIZ 22

MATH 200
November 13, 2025

1. (6 points) $\int \frac{1}{1+x^2}\,dx = \boxed{\tan^{-1}(x) + C}$ because $D_x\left[\tan^{-1}(x) + C\right] = \frac{1}{1+x^2}$

2. (7 points) The graph of a function $f(x)$ has a y-intercept of 7, and the tangent line to the graph at any point $(x, f(x))$ has slope $m = x^3 + 3x + 4$. Find the function $f(x)$.

Know $f'(x) = x^3 + 3x + 4$, so

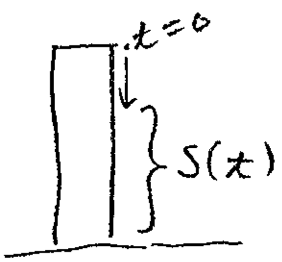
$$f(x) = \int x^3 + 3x + 4 \, dx = \frac{x^4}{4} + \frac{3}{2}x^2 + 4x + C$$

Also know $f(0) = 7$, so

$$7 = f(0) = \frac{0^4}{4} + \frac{3}{2}0 + 4\cdot 0 + C \Rightarrow C = 7$$

<u>Answer</u> $\boxed{f(x) = \frac{x^4}{4} + \frac{3}{2}x^2 + 4x + 7}$

3. (7 points) A rock is dropped from the top of a tower, and has a constant acceleration of -32 feet per second per second. It hits the ground after 10 seconds. Use calculus to find the height of the tower. (You may assume that its velocity is zero the instant it's dropped.)



Say the rock is dropped at time $t=0$ and its height above ground at time t is $S(t)$. Then the height of the tower will be $S(0)$.

Strategy: Find $S(t)$ and compute $S(0)$.

$v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C$.

Know $0 = v(0) = -32\cdot 0 + C \Rightarrow C = 0 \Rightarrow \boxed{v(t) = -32t}$

$s(t) = \int v(t)\,dt = \int -32t\,dt = -16t^2 + C$

$\Rightarrow \boxed{s(t) = -16t^2 + C}$, but $0 = s(10) = -16\cdot 10^2 + C$

$\Rightarrow C = 1600 \Rightarrow \boxed{s(t) = -16t^2 + 1600}$ $\boxed{\text{Tower is } s(0) = 1600' \text{ tall}}$