Name: ______________________________

QUIZ 22

MATH 200
April 19, 2022

1. (6 points) $\displaystyle\int \frac{\cos(x)+\sec(x)}{\cos(x)}\,dx = \int \left(\frac{\cos(x)}{\cos(x)} + \frac{\sec(x)}{\cos(x)}\right)dx = \int \left(1 + \frac{1}{\cos(x)}\sec x\right)dx =$

$$= \int (1+\sec(x)\sec(x))\,dx = \int \left(1+\sec^2(x)\right)dx = \boxed{x+\tan(x)+C}$$

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = 12x^2 - 6x + 1$ and $f(1) = 5$. Find $f(x)$.

$f(x) = \displaystyle\int \left(12x^2 - 6x + 1\right)dx = 12\frac{x^3}{3} - 6\frac{x^2}{2} + x + C = 4x^3 - 3x^2 + x + C$

So $f(x) = 4x^3 - 3x^2 + x + C$

We know $5 = f(1) = 4\cdot 1^3 - 3\cdot 1^2 + 1 + C = 2 + C$

Thus $5 = 2 + C$, so $C = 3$.

Therefore $\boxed{f(x) = 4x^3 - 3x^2 + x + 3}$

3. (7 points) A rock, propelled straight down from the top of a bridge over a river at time $t = 0$ seconds has a velocity of $v(t) = -32t - 5$ feet per second at time t. The rock hits water with a velocity of -69 feet per second. How high is the bridge?

First, we can find the instant the rock hits water by solving the equation $v(t) = -69$.

$$\begin{aligned} v(t) &= -69 \\ -32t - 5 &= -69 \\ = 32t &= 64 \\ t &= 2 \end{aligned}$$

Thus the rock hits the water precisely at $t = 2$ seconds.

Let $s(t)$ be the rock's height above the water at time t. So $s(2) = 0$ (as the rock hits water when t=2), and $s(0)$ is the height of the bridge.

Thus to find the height of the bridge we can find $s(t)$ and then compute $s(0)$.

We know $s(t) = \displaystyle\int v(t)\,dt = \int(-32t-5)\,dt = 16t^2 - 5t + C$ .... Thus $\boxed{s(t) = -16t^2 - 5t + C}$

To find C, recall that $0 = s(2) = -16\cdot 2^2 - 5\cdot 2 + C = -74 + C$, so $C = 74$, so $\boxed{s(t) = -16t^2 - 5t + 74}$

Answer: The height of the bridge is $s(0) = 74$ feet

Name: ______________________________ QUIZ 22 MATH 200

April 19, 2022

1. (6 points) $\int x^2\left(x^3+9x+18\right)\,dx = \int \left(x^5+9x^3+18x^2\right)\,dx = \frac{x^6}{6}+9\frac{x^4}{4}+18\frac{x^3}{3}+C =$

$$= \boxed{\frac{1}{6}x^6+\frac{9}{4}x^4+6x^3+C}$$

2. (7 points) Suppose $f(x)$ is a function for which $f'(x) = 9x^2+4x-8$ and $f(-1) = 1$. Find $f(x)$.

$f(x) = \int \left(9x^2+x-8\right)\,dx = 9\frac{x^3}{3}+4\frac{x^2}{2}-8x+C = 3x^3+2x^2-8x+C$

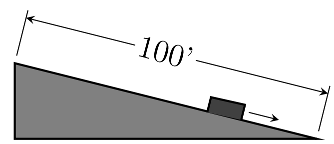
So $f(x) = 3x^3+2x^2-8x+C$

We know $1 = f(-1) = 3(-1)^3+2(-1)^2-8(-1)+C = -3+2+8+C = 7+C$

Thus $1 = 7+C$, so $C = -6$.

Therefore $\boxed{f(x) = 3x^3+2x^2-8x-6}$

3. (7 points) A block sliding down a 100-foot-long ramp has a constant acceleration of 2 feet per second per second. Its initial velocity (at the very top of the ramp) is 15 feet per second. How long does it take for the block to slide down the ramp.



Say the block starts sliding down the ramp at time $t = 0$ seconds.

Let $s(t)$ be the distance the block as come at time t. Thus at time $t = 0$, it has gone $s(0) = 0$ feet.

Velocity at time t is $v(t) = \int a(t)\,dt = \int 2\,dt = 2t+C$, that is, $v(t) = 2t+C$.

We know that $v(0) = 15$, so $15 = v(0) = 2\cdot 0+C$. Hence $C = 15$, so $\boxed{v(t) = 2t+15}$

Position at time t is $s(t) = \int v(t)\,dt = \int (2t+15)\,dt = t^2+15t+C$, that is, $s(t) = t^2+15t+C$.

We know that $0 = s(0) = 0^2+15\cdot 0+C$, so $C = 0$. Hence $\boxed{s(t) = t^2+15t}$

The block has reached the bottom at the time t for which $s(t) = 100$ (feet).

To find this t we need to solve the equation

$$\begin{aligned} s(t) &= 100 \\ t^2+15t &= 100 \\ t^2+15t-100 &= 0 \\ (t+20)(t-5) &= 0 \end{aligned}$$

The solutions are $t = -20$ and $t = 5$. We are assuming time t is *positive* so the answer is:

$\boxed{\text{It takes the block 5 seconds to slide down the ramp.}}$