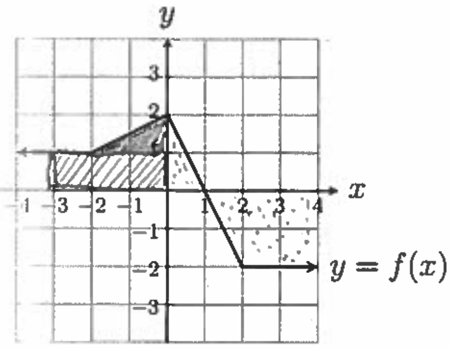
Name: Richard

QUIZ 23 ♡

MATH 200
December 6, 2021

1. Answer the questions about the function $f(x)$ graphed below.



(a) $\int_{-3}^{0} f(x)\,dx =$  +  $= 3 + \frac{1}{2}2\cdot 1 = \boxed{4}$

(b) $\int_{0}^{4} f(x)\,dx = A_{up} - A_{down} = 1 - 5 = \boxed{-4}$

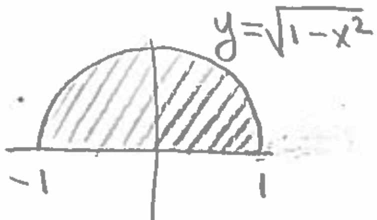
(c) Suppose $\int_0^7 f(x)\,dx = 10$. Find $\int_4^7 f(x)\,dx$.

$$\int_0^7 f(x)\,dx = \int_0^4 f(x)\,dx + \int_4^7 f(x)\,dx$$

$$10 = -4 + \int_4^7 f(x)\,dx$$

$$\boxed{\int_4^7 f(x)\,dx = 14}$$

2. Find $\int_{-1}^{1} 3\sqrt{1-x^2}\,dx$ by considering area.



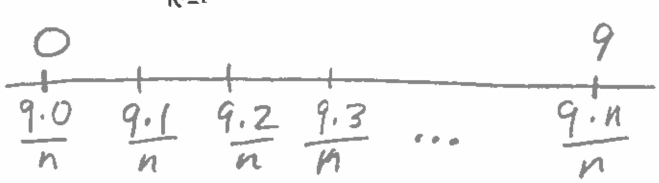
$$\int_{-1}^{1}\sqrt{1-x^2}\,dx = \frac{1}{2}\pi\cdot 1^2$$

$$\int_{-1}^{1} 3\sqrt{1-x^2}\,dx$$

$$= 3\cdot\int\sqrt{1-x^2}\,dx$$

$$= 3\cdot\frac{1}{2}\pi\cdot 1^2 = \boxed{\frac{3\pi}{2}}$$

3. Write $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\sqrt{2+\frac{9k}{n}}\,\frac{9}{n}$ as a definite integral.



Let $\Delta x = \frac{9-0}{n} = \frac{9}{n}$

Let $x_k = k\Delta x = \frac{9k}{n}$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{2+\frac{9k}{n}}\,\frac{9}{k} = \lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{2+x_k}\,\Delta x = \boxed{\int_0^9\sqrt{2+x}\,dx}$$

Also, if $x_k = 2+\frac{9k}{n}$ the integral is $\int_2^{11}\sqrt{x}\,dx$

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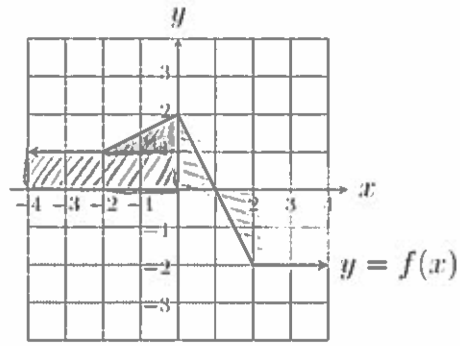
1. Answer the questions about the function $f(x)$ graphed below.



(a) $\displaystyle\int_{-1}^{0} f(x)\,dx =$ (rectangle) + (triangle) $= 4 + \frac{1}{2}2\cdot 1$
$= \boxed{5}$

(b) $\displaystyle\int_{0}^{2} f(x)\,dx =$ Aup − Adown $= \boxed{0}$

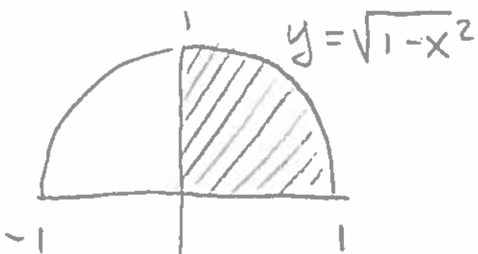
(c) Suppose $\displaystyle\int_{0}^{7} f(x)\,dx = 10$. Find $\displaystyle\int_{2}^{7} f(x)\,dx$.

$$\int_0^7 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^7 f(x)\,dx$$

$$10 = 0 + \int_2^7 f(x)\,dx$$

$$\therefore \boxed{\int_2^7 f(x)\,dx = 10}$$

2. Find $\displaystyle\int_0^1 3\sqrt{1-x^2}\,dx$ by considering area.



Area of circle πr^2

$$\int_0^1 3\sqrt{1-x^2}\,dx$$

$$= 3\int_0^1 \sqrt{1-x^2}\,dx$$

$$= 3\,\frac{1}{4}\pi\cdot 1^2 = \boxed{\frac{3\pi}{4}}$$

3. Write $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{2+\frac{8k}{n}}\,\frac{8}{n}$ as a definite integral.

0 ... 8

$\frac{8\cdot 0}{n}$ $\frac{8\cdot 2}{n}$ $\frac{8\cdot 3}{n}$ $\frac{8\cdot 4}{n}$ ... $\frac{8n}{n}$

Let $\Delta x = \frac{8-0}{n} = \frac{8}{n}$

and $x_k = \frac{8k}{n}$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{2+\frac{8k}{n}}\,\frac{8}{n} = \lim_{n\to\infty}\sqrt{2+x_k}\,\Delta x = \boxed{\int_0^8 \sqrt{2+x}\,dx}$$

Also if $x_k = 2+\frac{8k}{n}$ this is $\displaystyle\int_2^{10}\sqrt{x}\,dx$

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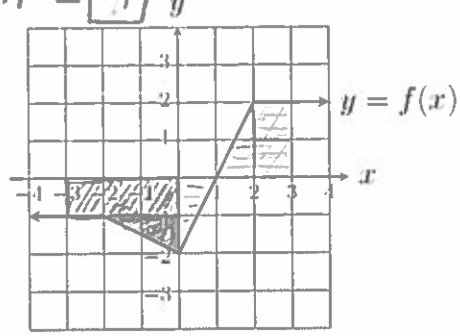
1. Answer the questions about the function $f(x)$ graphed below.



(a) $\int_{-3}^{0} f(x)\,dx =$ (rectangle) + (triangle) $= 3 + \frac{1}{2} 2 \cdot 1 = \boxed{-4}$

(b) $\int_{0}^{3} f(x)\,dx = A_{up} - A_{down} = \boxed{2}$

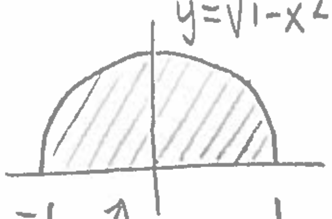
(c) Suppose $\int_{0}^{7} f(x)\,dx = 3$. Find $\int_{3}^{7} f(x)\,dx$.

$$\int_0^7 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^7 f(x)\,dx$$

$$3 = 2 + \int_3^7 f(x)\,dx$$

$$\therefore \boxed{\int_3^7 f(x)\,dx = 1}$$

2. Find $\int_{-1}^{1} 5\sqrt{1-x^2}\,dx$ by considering area.



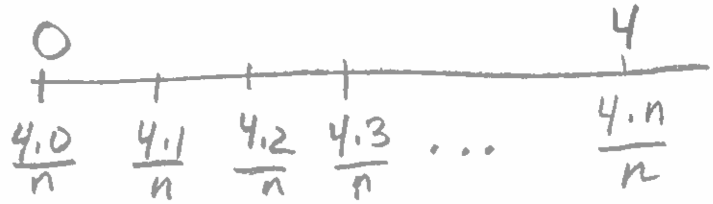
$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \frac{1}{2}\pi \cdot 1^2$$

$$\int_{-1}^{1} 5\sqrt{1-x^2}\,dx = 5\int_{-1}^{1} \sqrt{1-x^2}\,dx$$

$$= 5 \cdot \frac{1}{2}\pi \cdot 1^2$$

$$= \boxed{\frac{5\pi}{2}}$$

3. Write $\lim_{n\to\infty} \sum_{k=1}^{n} \ln\left(1+\frac{4k}{n}\right)\frac{4}{n}$ as a definite integral.



Let $\Delta x = \frac{4-0}{n}$

Let $x_k = \frac{4k}{n}$

$$\lim_{n\to\infty} \ln\left(1+\frac{4k}{n}\right)\frac{4}{n} = \lim_{n\to\infty} \ln(1+x_k)\Delta x = \boxed{\int_0^4 \ln(1+x)\,dx}$$

Also, if $x_k = 1+\frac{4k}{n}$ integral is $\boxed{\int_1^5 \ln(x)\,dx}$

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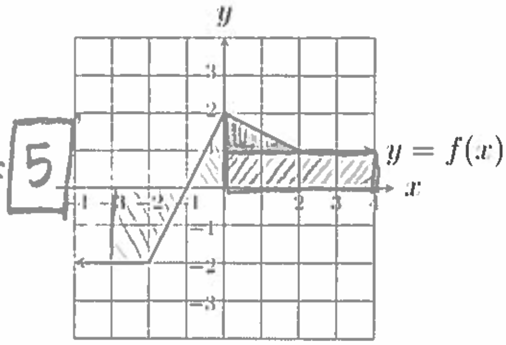
1. Answer the questions about the function $f(x)$ graphed below.



(a) $\int_{-3}^{0} f(x)\,dx = A_{up} - A_{down} = \boxed{-2}$

(b) $\int_{0}^{4} f(x)\,dx = \;\; + \;\; = 4 + \frac{1}{2}\,2\cdot 1 = \boxed{5}$

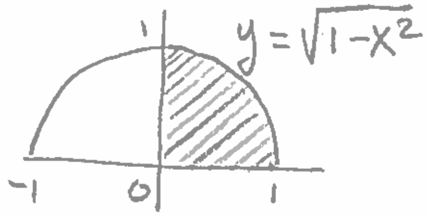
(c) Suppose $\int_0^7 f(x)\,dx = 10$. Find $\int_4^7 f(x)\,dx$.

$$\int_0^7 f(x)\,dx = \int_0^4 f(x)\,dx + \int_4^7 f(x)\,dx$$

$$10 = 5 + \int_4^7 f(x)\,dx$$

$$\therefore \boxed{\int_4^7 f(x)\,dx = 5}$$

2. Find $\int_0^1 5\sqrt{1-x^2}\,dx$ by considering area.



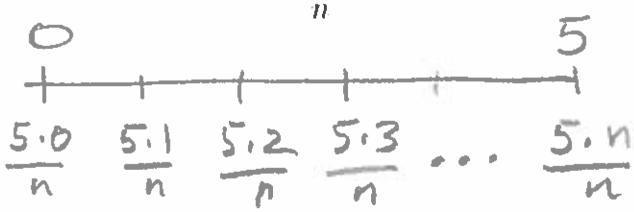
$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{1}{4}\pi\cdot 1^2 = \frac{\pi}{4}$$

$$\int_0^1 5\sqrt{1-x^2}\,dx$$

$$= 5\int \sqrt{1-x^2}\,dx$$

$$= 5\,\frac{1}{4}\pi\cdot 1^2 = \boxed{\frac{5\pi}{4}}$$

3. Write $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{2+\frac{5k}{n}}\cdot\frac{5}{n}$ as a definite integral.



Let $\Delta x = \frac{5-0}{n} = \frac{5}{n}$

Let $x_k = \frac{5k}{n}$

$$\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{2+\frac{5k}{n}}\,\frac{5}{n} = \lim_{n\to\infty} \frac{1}{2+x_k}\Delta x = \boxed{\int_0^5 \frac{1}{2+x}\,dx}$$

Also if $x_k = 2 + \frac{5k}{n}$ the integral is $\boxed{\int_2^7 \frac{1}{x}\,dx}$