Name: Richard

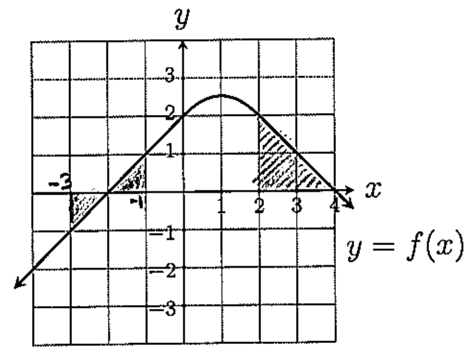
Quiz 23

MATH 200
November 28, 2023

1. Answer the questions about the function $f(x)$ graphed below.

(a) $\int_{-3}^{-1} f(x)\,dx = A_{up} - A_{down} = \frac{1}{2}1\cdot 1 - \frac{1}{2}1\cdot 1 = \boxed{0}$

(b) $\int_{4}^{2} f(x)\,dx = -\int_{2}^{4} f(x)\,dx = -\frac{1}{2}(2)(2) = \boxed{-2}$



(c) $\int_{-2}^{0} f(x)\,dx = \text{area} = \frac{1}{2}2\cdot 2 = \boxed{2}$

(d) Suppose $\int_{0}^{2} f(x)\,dx = 4.7$. Find $\int_{-2}^{4} f(x)\,dx$.

$$\int_{-2}^{4} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx = 2 + 4.7 + 2$$
$$= \boxed{8.7}$$

(e) $\lim_{n\to\infty} \sum_{k=1}^{n} f\left(-3 + \frac{2k}{n}\right)\frac{2}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x = \int_{-3}^{-1} f(x)\,dx = \boxed{0}$

$\Delta x = \frac{2}{n}$

$x_k = -3 + k\frac{2}{n} = -3 + k\Delta x$

$a = x_0 = -3 + 0\cdot\frac{2}{n} = -3$

$b = x_n = -3 + n\cdot\frac{2}{n} = -3 + 2 = -1$

2. Suppose for functions f and g we have: $\int_{1}^{4} f(x)\,dx = 1$, $\int_{4}^{6} f(x)\,dx = 2$, $\int_{1}^{6} g(x)\,dx = 3$.

Find $\int_{1}^{6} (2f(x) + g(x))\,dx$ $= \int_1^6 2f(x)\,dx + \int_1^6 g(x)\,dx$

$= 2\int_1^6 f(x)\,dx + \int_1^6 g(x)\,dx$

$= 2\left(\int_1^4 f(x)\,dx + \int_4^6 f(x)\,dx\right) + \int_1^6 g(x)\,dx$

$= 2(1 + 2) + 3 = \boxed{9}$

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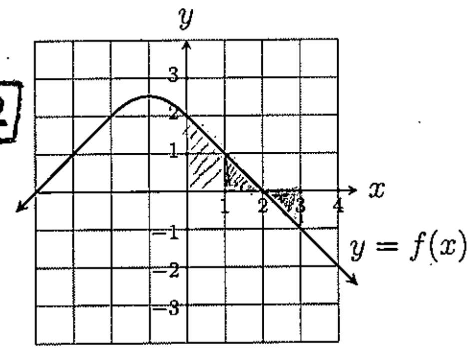
1. Answer the questions about the function $f(x)$ graphed below.

(a) $\int_1^3 f(x)\,dx = A_{up} - A_{down} = \frac{1}{2}1\cdot1 - \frac{1}{2}1\cdot1 = \boxed{0}$

(b) $\int_4^2 f(x)\,dx = -\int_2^4 f(x)dx = -(A_{up} - A_{down})$
$= -(0 - \frac{1}{2}2\cdot2) = \boxed{2}$



(c) $\int_0^1 f(x)\,dx = A_{up} - A_{down} = \left(\frac{3}{2} - 0\right) = \boxed{\frac{3}{2}}$

(d) Suppose $\int_{-2}^0 f(x)\,dx = 4.7$. Find $\int_{-2}^2 f(x)\,dx$.

$$\int_{-2}^2 f(x)\,dx = \int_{-2}^0 f(x)\,dx + \int_0^2 f(x)\,dx = 4.7 + \frac{1}{2}2\cdot2 = \boxed{6.7}$$

(e) $\lim_{n\to\infty}\sum_{k=1}^n f\left(\frac{k}{n}\right)\frac{1}{n} = \lim_{n\to\infty}\sum_{K=1}^n f(x_n)\Delta x = \int_0^1 f(x)dx = \boxed{\frac{3}{2}}$

$\Delta x = \frac{1}{n}$
$x_K = k\Delta x = \frac{k}{n}$
$a = x_0 = \frac{0}{n} = 0$
$b = x_n = n/n = 1$

2. Suppose for functions f and g we have: $\int_1^4 f(x)\,dx = 3$, $\int_4^6 f(x)\,dx = 2$, $\int_1^6 g(x)\,dx = 1$.

Find $\int_1^6 (5f(x) + g(x))\,dx$

$$= \int_1^6 5f(x)dx + \int_1^6 g(x)\,dx$$
$$= 5\int_1^6 f(x)\,dx + \int_1^6 g(x)dx$$
$$= 5\left(\int_1^4 f(x)dx + \int_4^6 f(x)dx\right) + \int_1^6 g(x)\,dx$$
$$= 5(3 + 2) + 1 = 5\cdot5 + 1 = \boxed{26}$$