Name: ______________________________

QUIZ 23

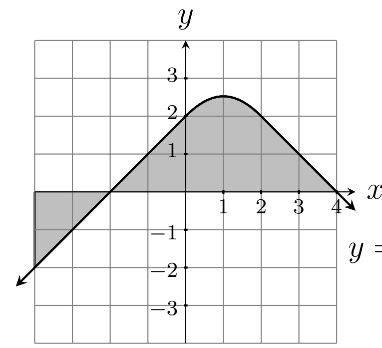
MATH 200
April 21, 2022

1. Answer the questions about the function $f(x)$ graphed below.

(a) $\int_{-2}^{0} f(x)\,dx = A_{up} - A_{down} = 2 - 0 = \boxed{2}$

(b) $\int_{0}^{-2} f(x)\,dx = -\int_{-2}^{0} f(x)\,dx = \boxed{-2}$

(c) $\int_{-4}^{-1} f(x)\,dx = A_{up} - A_{down} = \frac{1}{2} - 2 = \boxed{-\frac{3}{2}}$



$y = f(x)$

(d) Suppose $\int_{0}^{2} f(x)\,dx = 4.7$. Find $\int_{-2}^{2} f(x)\,dx$.

$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx = 2 + 4.7 = \boxed{6.7}$

(e) $\lim\limits_{n\to\infty} \sum_{k=1}^{n} f\left(2 + \frac{2k}{n}\right)\frac{2}{n} =$

Let $\Delta x = \frac{2}{n}$ and $x_k = 2 + k\Delta x = 2 + \frac{2k}{n}$.

Then $a = x_0 = 2 + \frac{2\cdot 0}{n} = 2$ and $b = x_n = 2 + \frac{2\cdot n}{n} = 4$

Thus $\lim\limits_{n\to\infty} \sum_{k=1}^{n} f\left(2+\frac{2k}{n}\right)\frac{2}{n} = \lim\limits_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x = \int_{2}^{4} f(x)\,dx = A_{up} - A_{down} = 2 - 0 = \boxed{2}$

2. Suppose for functions f and g we have: $\int_{1}^{4} f(x)\,dx = 1$, $\int_{4}^{6} f(x)\,dx = 3$, $\int_{1}^{6} g(x)\,dx = 4$.

Find $\int_{1}^{6} \big(f(x) + 2g(x)\big)\,dx$

Notice that $\int_{1}^{6} f(x)\,dx = \int_{1}^{4} f(x)\,dx + \int_{4}^{6} f(x)\,dx = 1 + 3 = 4$

Then $\int_{1}^{6} \big(f(x) + 2g(x)\big)\,dx = \int_{1}^{6} f(x)\,dx + \int_{1}^{6} 2g(x)\,dx = \int_{1}^{6} f(x)\,dx + 2\int_{1}^{6} g(x)\,dx = 4 + 2\cdot 4 = \boxed{12}$

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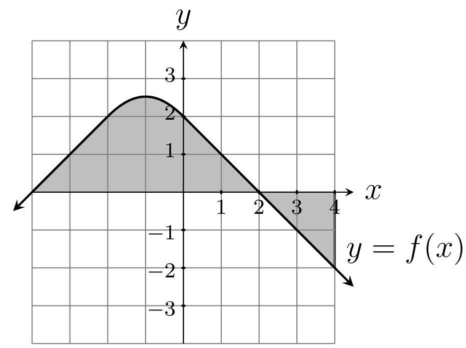
QUIZ 23

MATH 200
April 21, 2022

1. Answer the questions about the function $f(x)$ graphed below.

(a) $\displaystyle\int_1^4 f(x)\,dx = A_{up} - A_{down} = \frac{1}{2} - 2 = \boxed{-\frac{3}{2}}$

(b) $\displaystyle\int_4^1 f(x)\,dx = -\int_1^4 f(x)\,dx = \boxed{\frac{3}{2}}$



(c) $\displaystyle\int_0^2 f(x)\,dx = A_{up} - A_{down} = 2 - 0 = \boxed{2}$

(d) Suppose $\displaystyle\int_{-2}^0 f(x)\,dx = 4.7$. Find $\displaystyle\int_{-2}^2 f(x)\,dx$.

$\displaystyle\int_{-2}^2 f(x)\,dx = \int_{-2}^0 f(x)\,dx + \int_0^2 f(x)\,dx = 4.7 + 2 = \boxed{6.7}$

(e) $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} f\left(\frac{2k}{n}\right)\frac{2}{n} =$

Let $\Delta x = \dfrac{2}{n}$ and $x_k = k\Delta x = \dfrac{2k}{n}$.

Then $a = x_0 = \dfrac{2\cdot 0}{n} = 0$ and $b = x_n = \dfrac{2\cdot n}{n} = 2$

Thus $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} f\left(\frac{2k}{n}\right)\frac{2}{n} = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,\Delta x = \int_0^2 f(x)\,dx = A_{up} - A_{down} = 2 - 0 = \boxed{2}$

2. Suppose for functions f and g we have: $\displaystyle\int_1^4 f(x)\,dx = -1, \quad \int_4^6 f(x)\,dx = 2, \quad \int_1^6 g(x)\,dx = 3.$

Find $\displaystyle\int_1^6 \big(f(x) + 5g(x)\big)\,dx$

Notice that $\displaystyle\int_1^6 f(x)\,dx = \int_1^4 f(x)\,dx + \int_4^6 f(x)\,dx = -1 + 2 = 1$

Then $\displaystyle\int_1^6 \big(f(x) + 5g(x)\big)\,dx = \int_1^6 f(x)\,dx + \int_1^6 5g(x)\,dx = \int_1^6 f(x)\,dx + 5\int_1^6 g(x)\,dx = 1 + 5\cdot 3 = \boxed{16}$